Module 2 Notes for

"DIGITAL COMMUNICATION"
[BEC503]

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VISION

To produce competent engineering professionals in the field of Electronics and Communication Engineering by imparting value based quality technical education to meet the societal needs and to develop socially responsible citizens.

MISSION

M1: To provide strong fundamentals and technical skills in the field of Electronics and Communication Engineering through effective teaching learning process.

M2: Enhancing employability of the students by providing skills in the fields of VLSI, Embedded systems, Signal processing, etc., through Centre of Excellence.

M3: Encourage the students to participate in co-curricular and extra-curricular activities that creates a spirit of social responsibility and leadership qualities.

Program Specific Outcomes (PSOs)

After Successful Completion of Electronics and Communication Engineering Program Students will be able to

1. Apply fundamental knowledge of core. Electronics and Communication Engineering in the analysis, design and development of Electronics Systems as well as to interpret and synthesize experimental data leading to valid conclusions.
2. Exhibit the skills gathered to analyze, design, develop software applications and hardware products in the field of embedded systems and allied areas.

Program Educational Objectives (PEOs)

PEO1: Graduates exhibit their innovative ideas and management skills to meet the day to day technical challenges.

PEO2: Graduates utilize their knowledge and skills for the development of optimal solutions to the problems in the field of Electronics and Communication Engineering..

PEO3: Graduates exhibit good interpersonal skills, leadership qualities and adapt themselves for life-long Learning

DIGITAL COMMUNICATION		Semester	5
Course Code	BEC503	CIE Marks	50
Teaching Hours/Week (L:T:P: S)	4:0:0:0	SEE Marks	50
Total Hours of Pedagogy	50 Hours	Total Marks	100
Credits	04	Exam Hours	3 Hours
Examination type (SEE)	Theory		

Course objectives:

- Understand the concept of signal processing of digital data and signal conversion to symbols at the transmitter and receiver.
- Compute performance metrics and parameters for symbol processing and recovery in ideal and corrupted channel conditions.
- Understand the principles of spread spectrum communications.
- Understand the basic principles of information theory and various source coding techniques.
- Build a comprehensive knowledge about various Source and Channel Coding techniques.
- Discuss the different types of errors and error detection and controlling codes used in the communication channel.
- Understand the concepts of convolution codes and analyze the code words using time domain and transform domain approach.

Teaching-Learning Process (General Instructions)

These are sample Strategies, which teachers can use to accelerate the attainment of the various course outcomes.

1. Lecture method (L) does not mean only the traditional lecture method, but a different type of teaching method may be adopted to develop the outcomes.
2. Arrange visits to nearby PSUs such as BHEL, BEL, ISRO, etc., and small-scale communication industries.
3. Show Video/animation films to explain the functioning of various modulation techniques, Channel, and source coding.
4. Encourage collaborative (Group) Learning in the class
5. Ask at least three HOTS (Higher-order Thinking) questions in the class, which promotes critical thinking
6. Adopt Problem Based Learning (PBL), which fosters students' Analytical skills, develop thinking skills such as the ability to evaluate, generalize & analyze information rather than simply recall it.
7. Topics will be introduced in multiple representations.
8. Show the different ways to solve the same problem and encourage the students to come up with their own creative ways to solve them.
9. Discuss how every concept can be applied to the real world - and when that's possible, it helps improve the students' understanding.

Module-1

Bandpass Signals to Equivalent Lowpass: Hilbert Transform, Pre-envelopes, Complex envelopes of Band-pass Signals, Canonical Representation of Bandpass signals.

Signalling over AWGN Channels- Introduction, Geometric representation of signals, Gram- Schmidt Orthogonalization procedure, Conversion of the continuous AWGN channel into a vector channel , Optimum receivers using coherent detection: ML Decoding, Correlation receiver, matched filter receiver.

Module-2

Digital Modulation Techniques: Phase shift Keying techniques using coherent detection: generation, detection and error probabilities of BPSK and QPSK, M–ary PSK, M–ary QAM. Frequency shift keying techniques using Coherent detection: BFSK generation, detection and error probability. BFSK using Noncoherent Detection, Differential Phase Shift Keying.

Module-3

Information theory: Introduction, Entropy, Source Coding Theorem, Lossless Data Compression Algorithms, Discrete Memoryless Channels, Mutual Information, Channel capacity, Channel Coding Theorem, Information Capacity Law (Statement).

Module-4

Error Control Coding: Error Control Using Forward error Correction,

Linear Block Codes: Definitions, Matrix Descriptions, Syndrome and its properties, Minimum distance Considerations, Syndrome Decoding, Hamming Codes.

Cyclic Codes: Properties, Generator and Parity Check Polynomial and matrices, Encoding, Syndrome computation, Examples.

Module-5

Convolutional Codes: Convolutional Encoder, Code tree, Trellis Graph and State graph, Recursive systematic Convolutional codes, Optimum decoding of Convolutional codes, Maximum Likelihood Decoding of Convolutional codes: The Viterbi Algorithm, Examples.

Course outcome (Course Skill Set)

At the end of the course, the student will be able to :

1. Apply the concept of signal conversion to vectors in communication transmission and receiption.
2. Perform the mathematical analysis of digital communication systems for different modulation techniques.
3. Apply the Source coding and Channel coding principles for the discrete memoryless channels.
4. Compute the codewords for the error correction and detection of a digital data using Linear Block Code, Cyclic Codes and Convolution Codes.
5. Design encoding and decoding circuits for Linear Block Code, Cyclic Codes and Convolution Codes.

MODULE - 02

DIGITAL MODULATION TECHNIQUES

- In modulation we have m(t), c(t) and s(t)
- m(t) is the message signal which has low frequency. If there is a variation in the message signal then there will be a variation in the information that is taken
- c(t) is a carrier signal which has high frequency. We can vary this signal according to the message signal with a parameters like amplitude, frequency and phase
- S(t) is the Output signal

★ BPS (Band Pass filter) ⟶ Purest form of the message signal

★ ASK (Amplitude shift key) ⟶ Vary amplitude, frequency and phase is constant

★ PSK (Phase shift key) ⟶ Vary phase, frequency and amplitude is constant

★ FSK (Frequency shift key) ⟶ Vary frequency, Amplitude and phase is constant

★ Detection

- Coherent: For this detection we use carriers for both transmitter and receiver. It is the process where the carrier signal should be synchronised (signal) in sense should be same in frequency and phase.

- <u>Non-coherent</u> : for this detection we use noise less channel

The carrier need not be synchronised, meaning that frequency and phase of a signal can be different.

ASK - It is also known as ON and OFF channel keying

ASK - It is non-linearity.

★ <u>BPSK (Binary phase shift key)</u>

In Binary PSK two signals are used to represent symbol 0 and symbol 1.

Symbol 1 is represented as $S_1(t)$

Symbol 0 is represented as $S_2(t)$

As we are using two signals. $\boxed{M=2}$ and $\boxed{N=1}$

for modulating the signal we need a carrier signal $c(t)$

$$c(t) = A(\cos 2\pi f_c t + \phi)$$

for transmitting

$$1 \longrightarrow \phi = 0 \; ; \; c(t) = A\cos 2\pi f_c t$$

$$0 \longrightarrow \phi = \pi \; ; \; c(t) = A(\cos 2\pi f_c t + \pi)$$

$$c(t) = -A\cos 2\pi f_c t$$

for transmitting 1 and 0

$$1 \longrightarrow S_1(t) = A\cos 2\pi f_c t$$

$$0 \longrightarrow S_2(t) = -A\cos 2\pi f_c t$$

$$S_2(t) = -S_1(t)$$

- $S_2(t)$ is dependent on $S_1(t)$.
- We have One independent signal ∴ The number of Orthonormal basis function used to represent the signal is 1.
- The value of $N=1$, the time period varies from $0 \leq t \leq T_b$
- The energy of the signal $S_1(t)$ in the bit period is given by

$$E_b = \int_0^{T_b} s_1^2(t)\,dt$$ Eb is energy per bit

W.K.T $S_1(t) = A^2 \cos^2 2\pi f_c t\,dt$

$\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$ or $\frac{1+\cos 2\theta}{2}$

So $$\boxed{E_b = \int_0^{T_b} A^2 \cos^2 2\pi f_c t\,dt}$$

$$E_b = \frac{A^2}{2}\left[\int_0^{T_b} dt + \int_0^{T_b} \cos 4\pi f_c t\,dt\right]$$

$$E_b = \frac{A^2}{2}\left[[t]_0^{T_b} + \left[\frac{\sin 4\pi f_c t}{\sin 4\pi f_c}\right]_0^{T_b}\right]$$

$$E_b = \frac{A^2}{2}\left[T_b + \cancel{\frac{\sin 4\pi f_c T_b}{4\pi f_c}}^{\,0}\right]$$

$$\boxed{E_b = \frac{A^2 T_b}{2}}$$

$$\boxed{A = \sqrt{\frac{2E_b}{T_b}}} \Rightarrow A\sqrt{\frac{T_b}{2}}$$

Substitute the value of A

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

The arthanarmal basic function $\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$

where E_1 is the energy of the signals $S_1(t)$.

So, $\phi_1(t) = \frac{\sqrt{\frac{2Eb}{Tb}} \cos 2\pi fct}{A\sqrt{\frac{Tb}{2}}}$

$$\phi_1(t) = \frac{\cancel{A} \cos 2\pi fct}{\cancel{A}\sqrt{\frac{Tb}{2}}}$$

$$\boxed{\phi_1(t) = \sqrt{\frac{2}{Tb}} \cos 2\pi fct}$$

Representing the signal in terms of arthonarmal basic function

$$\boxed{S_1(t) = \sqrt{Eb}\, \phi_1(t)}$$

$$\boxed{S_2(t) = -\sqrt{Eb}\, \phi_2(t)}$$

The vector $S_{ij} = \int_0^{Tb} S_i(t)\, \phi_j(t)\, dt$

S_{ij} is the vector representation of the signal

$i = 1, 2, ---- M$; $j = 1, ---- M$

$i = 1, 2$; $j = 1$

$$S_{11} = \int_0^{Tb} s_1(t)\, \phi_1(t) dt$$

$$S_{11} = \int_0^{Tb} \sqrt{Eb}\, \phi_1(t) \cdot \phi_1(t) dt$$

$$S_{11} = \int_0^{Tb} \sqrt{Eb}\, \phi_1^2(t)\, dt$$

from the property of arthanarmal basic function

$$\int_0^T \phi_1^2(t) \cdot dt = 1.$$

$$\boxed{S_{11} = \sqrt{E_b}}$$

Similarly

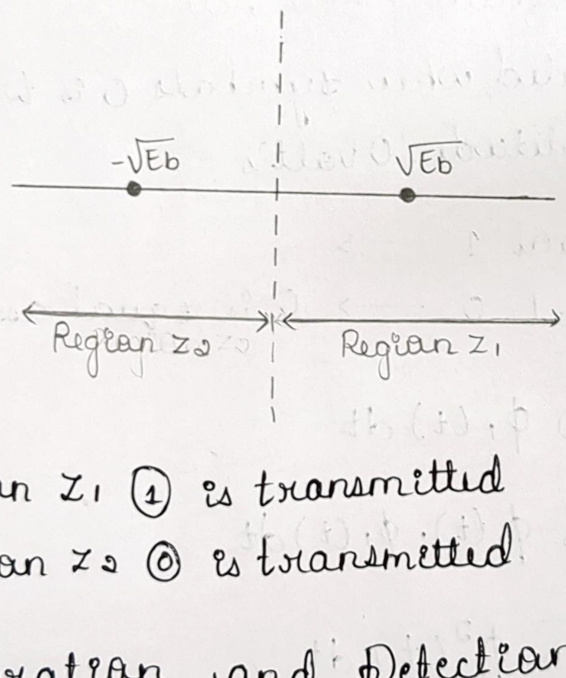
$$S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt$$

$$S_{21} = \int_0^{T_b} -\sqrt{E_b}\,\phi_1(t)\cdot\phi_1(t)\,dt$$

$$S_{21} = \int_0^{T_b} -\sqrt{E_b}\,\phi_1^2(t)\,dt$$

$$S_{21} = -\sqrt{E_b}$$

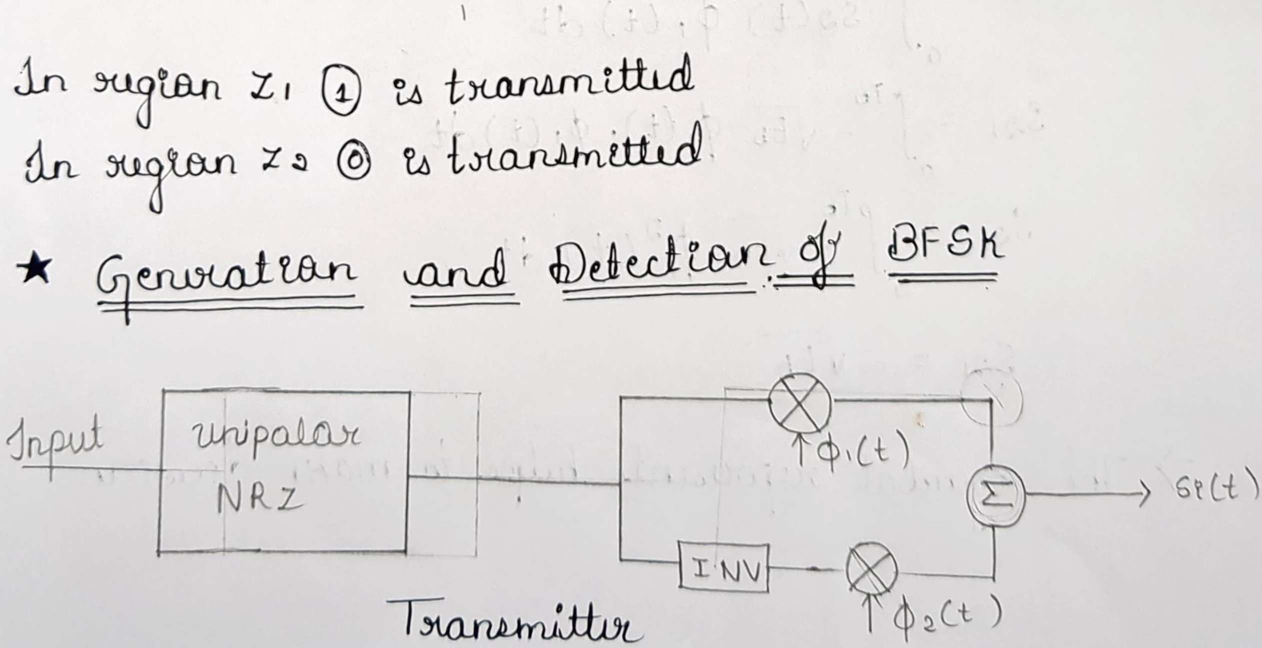
Constillation Diagram



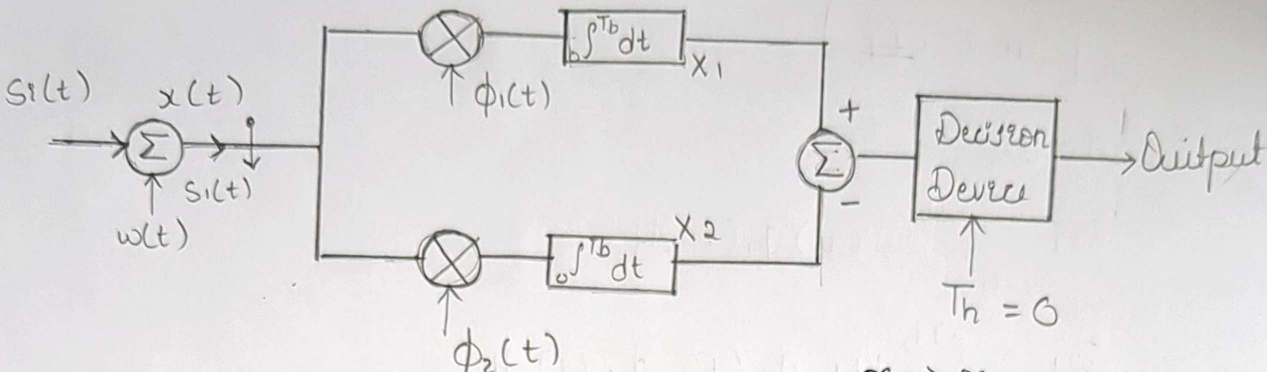
In region Z_1 (1) is transmitted

In region Z_2 (0) is transmitted

★ Generation and Detection of BFSK



Transmitter



Reciver

$x_1 > x_2 \longrightarrow 1 \quad 0-1$

$x_1 < x_2 \longrightarrow 0 \quad -1<0$

* <u>Transmitter</u>

The input is binary data

- Unipolar NRZ is given
- Unipolar is represented when symbols 1 is transmitting it is done with amplitude, 1v's, etc
- Unipolar is represented when symbals 0 is transmitting it is done with amplitude, 0 volt's

Transmitter symbol 1 $\longrightarrow$

symbal 0 $\longrightarrow$ 0 is ~~equal~~ assigned

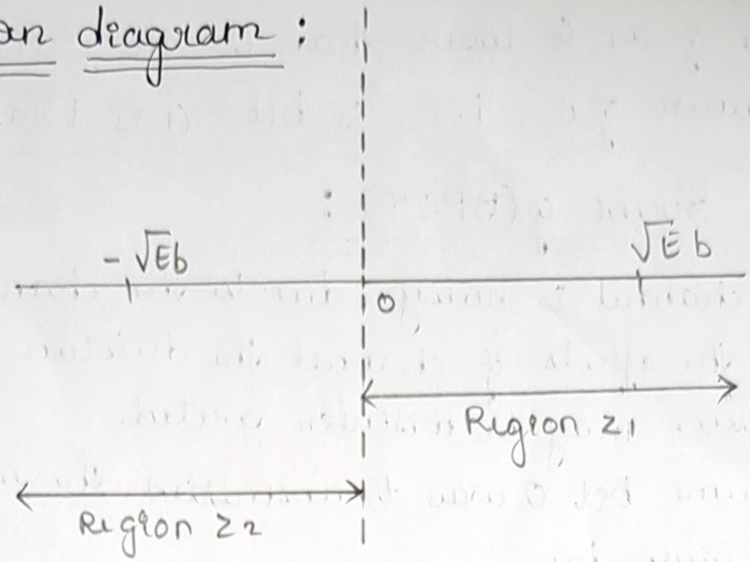
$\rightarrow$ ll^{ly} $\quad s_{21} = \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt$

$$s_{21} = \int_0^{T_b} -\sqrt{E_b}\,\phi_1(t).\,\phi_1(t)\,dt$$

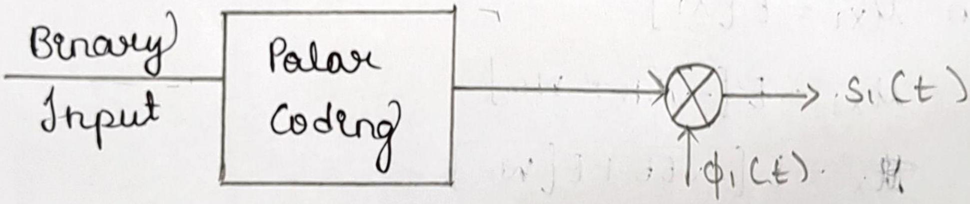
$$s_{21} = \int_0^{T_b} -\sqrt{E_b}\,\phi_1^2(t)\,dt$$

$$s_{21} = -\sqrt{E_b}$$

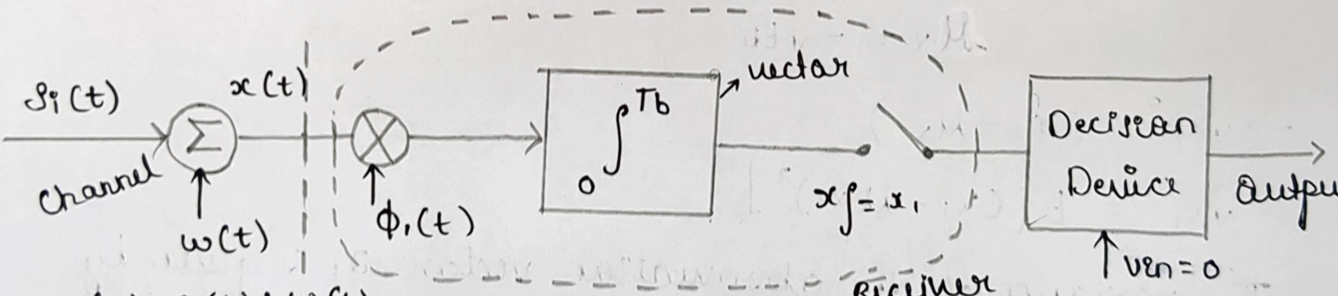
$\Rightarrow$ The symbal represents helps to make decision constellation diagram :



★ Generation and Detection of BPSK (BD, constellation diagram)



Transmitter



$x(t) = s_1(t) + w(t)$

→ symbol 1 ; $s_1(t) = \sqrt{E_b}\ \phi_1(t)$ → (assigning positive amplit

→ symbol 0 ; $s_2(t) = \sqrt{E_b}\ \phi_1(t)$ → (assigning negative amplitu

$$x_1 = \begin{cases} \sqrt{E_b} + W_1 \text{ ; Bit 1 Transmitted} \\ -\sqrt{E_b} + W_2 \text{ ; Bit 0 Transmitted} \end{cases}$$

→ The Output of integration is compared with the threshold value in the decision device, if the value of x_1 is greater a the decision goes to 1 that is bit 1 is Transmitted

If the value of x_1 is lesser than 0, the decision goes in the favour of 0. i.e is bit 0 is transmitted

★ **Probability Error of (BPSK):**

→ When the channel is noisy, due to the channel characteristics or the effects of channel the decision made by decision device may be misinterpreted.

→ If we assume bit 0 was transmitted the observation vector is given by

$$x_1 = -\sqrt{E_b} + W_1$$

→ The mean $\mu_{X_1} = E[X_1]$

$$\mu_{X_1} = E[-\sqrt{E_b} + W_1]$$

$$\mu_X = E[-\sqrt{E_b} + E[W_1]$$

$$\mu_X = E[-\sqrt{E_b}] + 0$$

$$\mu_X = -\sqrt{E_b}$$

→ $\sqrt{X_1}^2 = (x_1 - \mu_{X_1})$

$$\sqrt{X_1}^2 = E[(X_1 - \mu_{X_1})^2]$$

The variance of the observation vector x_1 is given by

$$\sqrt{x_1}^2 = E[(-\sqrt{E_b} + W_1 + \sqrt{E_b})^2]$$

$$\sqrt{x_1}^2 = E[W_1^2]$$

The variance of white noise is $\frac{N_0}{2}$

$$\therefore \sqrt{x_1}^2 = \frac{N_0}{2}$$

* The conditional probability density function of random variable x_1 is given by

$$f_{x_1}(x_1 | \text{Transmitted}) = \frac{1}{\sqrt{2\pi \sigma_{x_1}^2}} \exp\left[\frac{-(x_1 - \mu)^2}{2\sigma_{x_1}^2}\right]$$

→ The conditional probability density function of a random variable x_1 symbol was transmitted

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{\cancel{2}\pi \frac{N_0}{\cancel{2}}}} \exp\left[\frac{-(x+\sqrt{E_b})^2}{2 \times \frac{N_0}{2}}\right]$$

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x+\sqrt{E_b})^2}{N_0}\right]$$

→ when symbol 0 is transmitted an error will occur if $x_1 > 0$, where decision goes in the favour of 1

→ The probability of error of bit 0 is equal to probability that $x_1 > 0$ when symbol 0 was transmitted

$$\boxed{P_e(0) = P(x_1 > 0 / \text{symbol 0 Transmitted})}$$

→ The probability of error of bit 0 can be found by integrating the conditional probability density function

$$P_e(0) = \int_0^\infty f_{x_1}(x_1 / (0))\, dx_1$$

$$P_e(0) = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(x+\sqrt{E_b})^2}{N_0}\right] dx$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[\frac{-(x+\sqrt{E_b})^2}{N_0}\right] dx$$

Let

$$u = \frac{x+\sqrt{E_b}}{\sqrt{N_0}}$$

$$u \cdot \sqrt{N_0} = x_1 + \sqrt{E_b} \longrightarrow x_1 = 0 \,;\; u = \sqrt{\frac{E_b}{N_0}}$$

$$du \cdot \sqrt{N_0} = dx_1 \longrightarrow x_1 = \infty \,;\; u = \infty$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-u^2} \cdot du \cdot \sqrt{N_0}$$

$$P_e(0) = \frac{1}{\sqrt{\pi}\sqrt{N_0}} \times \sqrt{N_0} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-u^2} du$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-u^2} du$$

→ The error function of random variable is given by

$$\boxed{\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-z^2} dz}$$

$$\Rightarrow P_e(0) = \frac{1}{2} \times \frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-u^2} du$$

$$P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\therefore P_e(0) - P_e(1)$$

→ The total probability of error is equal to

$$P_e = P(0)\, P_e(0) + P(1)\, P_e(1)$$

$$= \frac{1}{2}\left[\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right]$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

★ Quadrature Phase Shift Keying

- It is variation of PSK
- It will be transmitting 4 symbols $M = 4$
- Number of bits / symbal $= \log_2 M$

$$= \log_2 4$$

$$= 2$$

- symbal duration $=$ bit duration $\times \log_2 M$

$$= T_b \times 2$$

$$= 2T_b$$

- will transmit group of zero's and one's (or) combination of 0's / 1's
- The QPSK transmitted signal is given by

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\pi/4\right] \quad \text{—(1)}$$

i varies from 1 to m

$S_1(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{\pi}{4}\right)$ when $i = 1$

$S_2(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{3\pi}{4}\right)$ when $i = 2$

$S_3(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{5\pi}{4}\right)$ when $i = 3$

$S_4(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{7\pi}{4}\right)$ when $i = 4$

we know that ; $\cos(a+b) = \cos a \cos b - \sin a \sin b$

$$S_i(t) = \sqrt{\frac{2E}{T}}\left(\cos 2\pi f_c t \cdot \cos(2i-1)\frac{\pi}{4} - \sin 2\pi f_c t \cdot \sin(2i-1)\frac{\pi}{4}\right)$$

we need 4 quadrants to represent a QPSK signal therefore The value of $\boxed{N = 2}$ (basic function) where $\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$

$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$$S_i(t) = \sqrt{E}\left(\cos(2i-1)\frac{\pi}{4} \cdot \phi_1(t) - \sin(2i-1)\frac{\pi}{4}\,\phi_2(t)\right)$$

$$S_i = \begin{bmatrix} \sqrt{E} \cos(2i-1)\pi/4 \\ \sqrt{E} \cos(2i-1)\pi/4 \end{bmatrix}$$

$i = 1$

$$S_i = \begin{bmatrix} \sqrt{E} & \frac{1}{\sqrt{2}} \\ -\sqrt{E} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$S_1 = \left(\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}\right)$$

$i = 2$

$$\begin{bmatrix} -\sqrt{E} & \frac{1}{\sqrt{2}} \\ \sqrt{E} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

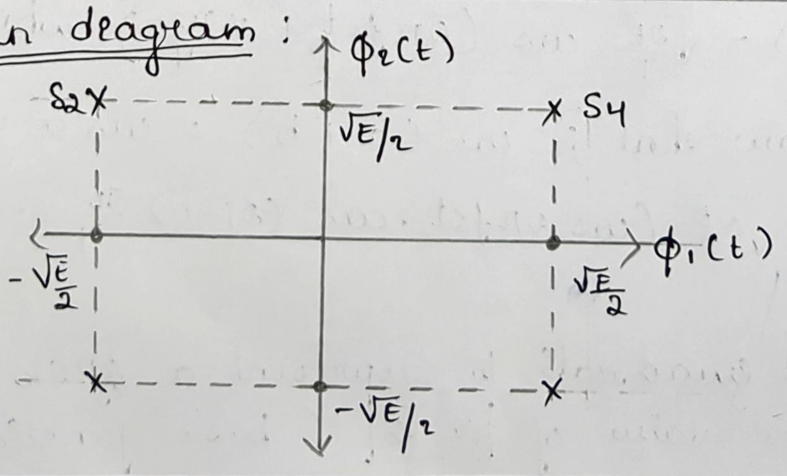
$$S_2 = \left(-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}\right)$$

$i = 3$, $S_3 = -\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}$

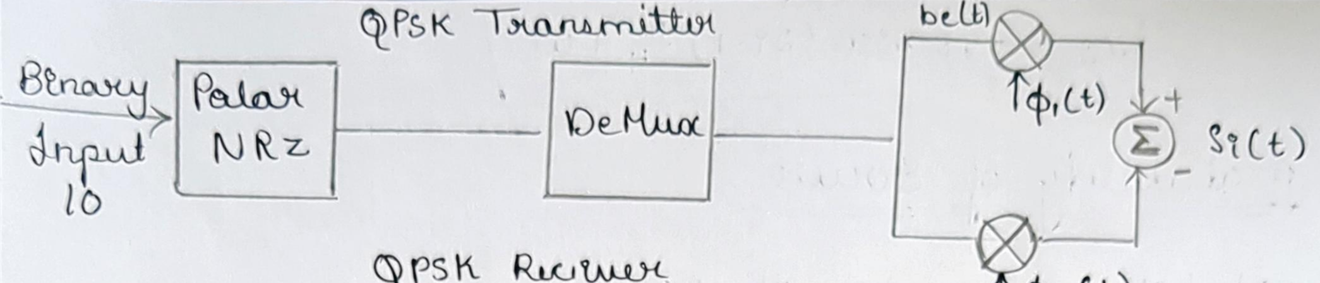
$i = 4$ $S_4 = \left(\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}\right)$

Binary digit		phase	Message coordinate
S_1	10	$\frac{\pi}{4}$	$\left[\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}\right]$
S_2	01	$\frac{3\pi}{4}$	$\left[-\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}\right]$
S_3	00	$\frac{5\pi}{4}$	$\left[-\sqrt{\frac{E}{2}}, -\sqrt{\frac{E}{2}}\right]$
S_4	11	$\frac{7\pi}{4}$	$\left[\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}}\right]$

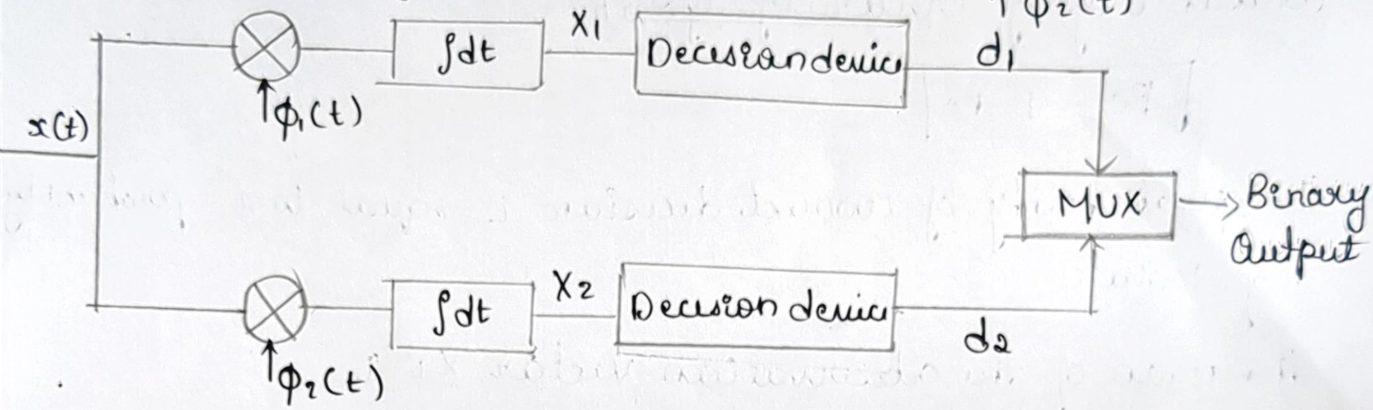
Constellation diagram :



QPSK Transmitter



QPSK Receiver



$$x_1 = \int_0^T x(t)\,\phi_1(t)\,d(t)$$

$$x_1 = \int_0^T (s_i(t) + W(t))\,\phi_1(t)\,dt$$

$$x_1 = \int_0^T \left\{\left[\sqrt{E}\left(\cos(2i-1)\pi/4 \cdot \phi_1(t)\right) - \sin(2i-1)\pi/4 \cdot \phi_2(t)\right] + w(t)\right\}\phi_1(t)\,dt$$

$$x_1 = \sqrt{E}\int_0^T \left(\cos(2i-1)\pi/4\, \phi_1^2(t) - \sin(2i-1)\frac{\pi}{4}\,\phi_1(t)\,\phi_2(t)\right]dt + w(t)\,\phi_1(t)\,dt$$

$$\boxed{x_1 = \sqrt{E}\cos(2i-1)\pi/4 + w}$$

$$x_2 = \int_0^T x(t)\,\phi_2(t)\,dt \Rightarrow x_2 = \int_0^T (s_i(t) + w(t)\,w_2(t)])\,dt$$

$$x_2 = \int_0^T \left\{\left[\sqrt{E}\left(\cos(2i-1)\pi/4\right)\phi_1(t) - \sin(2i-1)\frac{\pi}{4}\,\phi_2(t)\right] + w(t).\right\}\phi_1(t)\,dt$$

$$x_2 = \sqrt{E}\int_0^T \left[\cos(2i-1)\frac{\pi}{4}\,\phi_1(t).\phi_2(t) - \sin(2i-1)\frac{\pi}{4}\,\phi_2^2(t)\right]dt + \int_0^T w(t)\,\phi_2(t)\,dt$$

$$x_2 = \sqrt{E}\int_0^T -\sin(2i-1)\pi/4\,\phi_2^2(t)\,dt + \int_0^T w(t)\,\phi_2(t)\,dt$$

$$\boxed{x_2 = -\sqrt{E}\,\sin(2i-1)\frac{\pi}{4} + w}$$

★ **probability of Error**

probability of error of QPSK

$$\boxed{P_c = 1 - P_e}$$

The probability of correct decision is equal to 1 - probabity of Error

The mean of the observation vector X_1 is

$$x_1 = \sqrt{\frac{E}{2}} + w \;;\; x_1 > 0$$

$$x_2 = \sqrt{\frac{E}{2}} + w \;;\; x_2 > 0$$

So

$$\mu x_1 = E(x_1)$$

$$= E\left[\sqrt{\frac{E}{2}} + w\right]$$

$$\mu x_1 = \sqrt{\frac{E}{2}}$$

III^ly

$$\mu x_2 = E[x_2]$$

$$= E\left[\sqrt{\frac{E}{2}} + w\right]$$

$$\boxed{\mu x_2 = \sqrt{\frac{E}{2}}}$$

The probability of correct decision $(P_c) = P(x_1 > 0) * P(x_2 > 0)$

The conditional probability density function of a random Variable is given by

$$f_{x_1}(x_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x_1-\mu)^2}{2\sigma^2}\right]$$

The varience of white noise is $\sigma^2_{var} = E[x_1 - \mu x_1]$

$$= E\left[\sqrt{\tfrac{E}{2}} + w - \sqrt{\tfrac{E}{2}}\right]$$

$$= E[W]$$

$$\boxed{\sigma^2_{var} = \frac{N_0}{2}}$$

$$f_{x_1}(x_1) = \frac{1}{\sqrt{2\pi N_0/2}} \exp\left[\frac{-(x_1-\sqrt{\frac{E}{2}})^2}{2 N_0/2}\right]$$

$$f_{x_1}(x_1) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x_1-\sqrt{E/2})^2}{N_0}\right]$$

llly

$$f_{x_2}(x_2) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x_2-\sqrt{\frac{E}{2}})^2}{N_0}\right]$$

The probability that $x_1 > 0$ is given by

$$P(x_1>0) = \int_0^\infty \underbrace{f_{x_1}(x_1)\, dx_1}$$ → probability density function

$$P(x_1>0) = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(x_1-\sqrt{\frac{E}{2}})^2}{N_0}\right] dx_1$$

$$P(x_1>0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1-\sqrt{E/2})^2}{N_0}\right] dx_1$$

$$u = \frac{x_1 - \sqrt{E/2}}{N_0}$$

$$u\sqrt{N_0} = x_1 - \sqrt{E/2}$$

$$u\sqrt{N_0} = x_1 - \sqrt{\frac{E}{2}}$$

$$x_1 = 0 \Rightarrow u = -\sqrt{\frac{E}{2N_0}}$$

$$x_1 = \infty \Rightarrow u = \infty$$

$dx \sqrt{N_0} = dx_1$

$$P(x_1 > 0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E}{N_0}}}^{\infty} e^{-u^2} du \sqrt{N_0}$$

$$P(x_1 > 0) = \frac{1}{\sqrt{\pi N_0}} \times \sqrt{N_0} \int_{-\sqrt{\frac{E}{N_0^2}}}^{\infty} e^{-u^2} du$$

$$P(x_1 > 0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E}{2N_0}}}^{\infty} e^{-u^2} du$$

$$\operatorname{erfc}(-x) = 2 - \operatorname{erf} x$$

$$P(x_1 > 0) = \frac{1}{2} \times \frac{2}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} e^{-u^2} du$$

$$P(x_1 > 0) = \frac{1}{2} \operatorname{erfc}\left(-\sqrt{\frac{E}{2N_0}}\right)$$

$$= \frac{1}{2}\left[2 - \operatorname{erfc} \sqrt{\frac{E}{2N_0}}\right]$$

$$P(x_1 > 0) = 1 - \frac{1}{2} \operatorname{erf} \sqrt{\frac{E}{2N_0}}$$

$$P_c = 1 - P_e$$

$$P_e = 1 - P_C \quad \text{——} \quad (B)$$

$$P_c = \left[1 - \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E}{2N_0}}\right]^2$$

$$P_c = 1 + \frac{1}{4} \operatorname{erfc}^2 \sqrt{\frac{E}{2N_0}} - 2 \times \frac{1}{2} \operatorname{erf} \sqrt{\frac{E}{2N_0}}$$

$$P_c = 1 + \frac{1}{4} \operatorname{erfc}^2 \sqrt{\frac{E}{2N_0}} - \operatorname{erfc} \sqrt{\frac{E}{2N_0}}$$

Substitute the above equation in (B)

$$P_e = 1 - \left[1 + \frac{1}{4} \operatorname{erfc}^2\left[\sqrt{\frac{E}{2N_0}}\right) - \operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right]$$

$$P_e = -\frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) + \operatorname{erf}\sqrt{\frac{E}{2N_0}}$$

$$\operatorname{erfc}^2 = 0$$

$$\boxed{P_e = \operatorname{erfc}\sqrt{\frac{E}{2N_0}}} \longrightarrow \text{symbol error rate}$$

$$\boxed{P_e = \operatorname{erfc}\sqrt{\frac{2E_b}{2N_0}}} \quad E = 2E_b$$

$$T = 2T_b$$

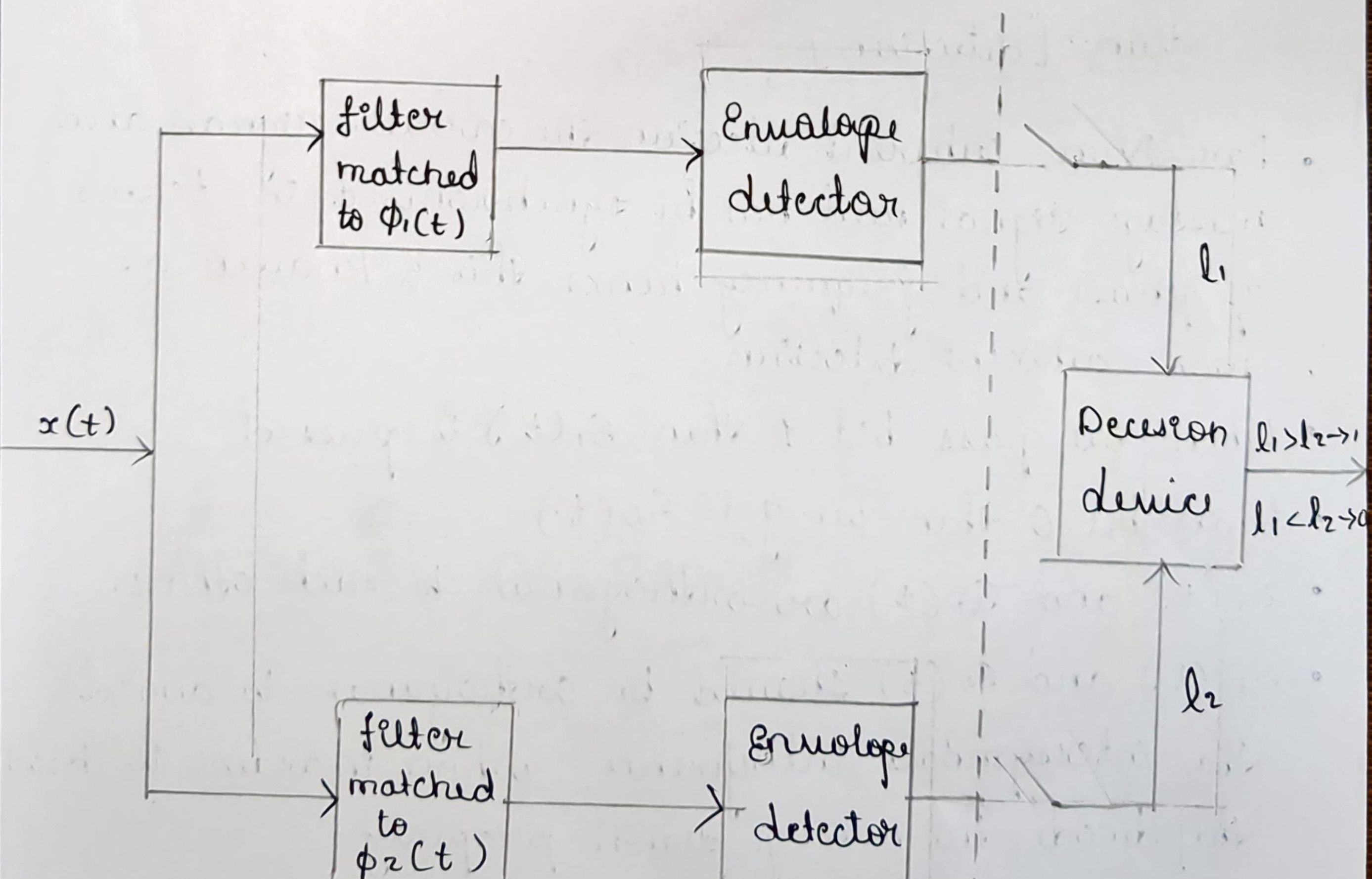
$$\boxed{P_e = \operatorname{erfc}\sqrt{\frac{2E_b}{N_0}}}$$

★ <u>Non-coherent detection / Non-Orthogonal detmodulation [detection]</u>

- For Non-coherent detection the carrier signal and message signal need not be synchronized in terms of phase and frequency hence this is known as Non-coherent detection.
- When we pass bit 1 then $S_1(t)$ is passed
- When bit 0 then we get $S_2(t)$
- $S_1(t)$ and $S_2(t)$ are orthogonal to each other.
- $S_1(t)$ and $S_2(t)$ should be orthogonal to avoid the intersymbols interference, where inorder to detect the/decode the message signal properly.
- The received signal have unknown phase shift and noise

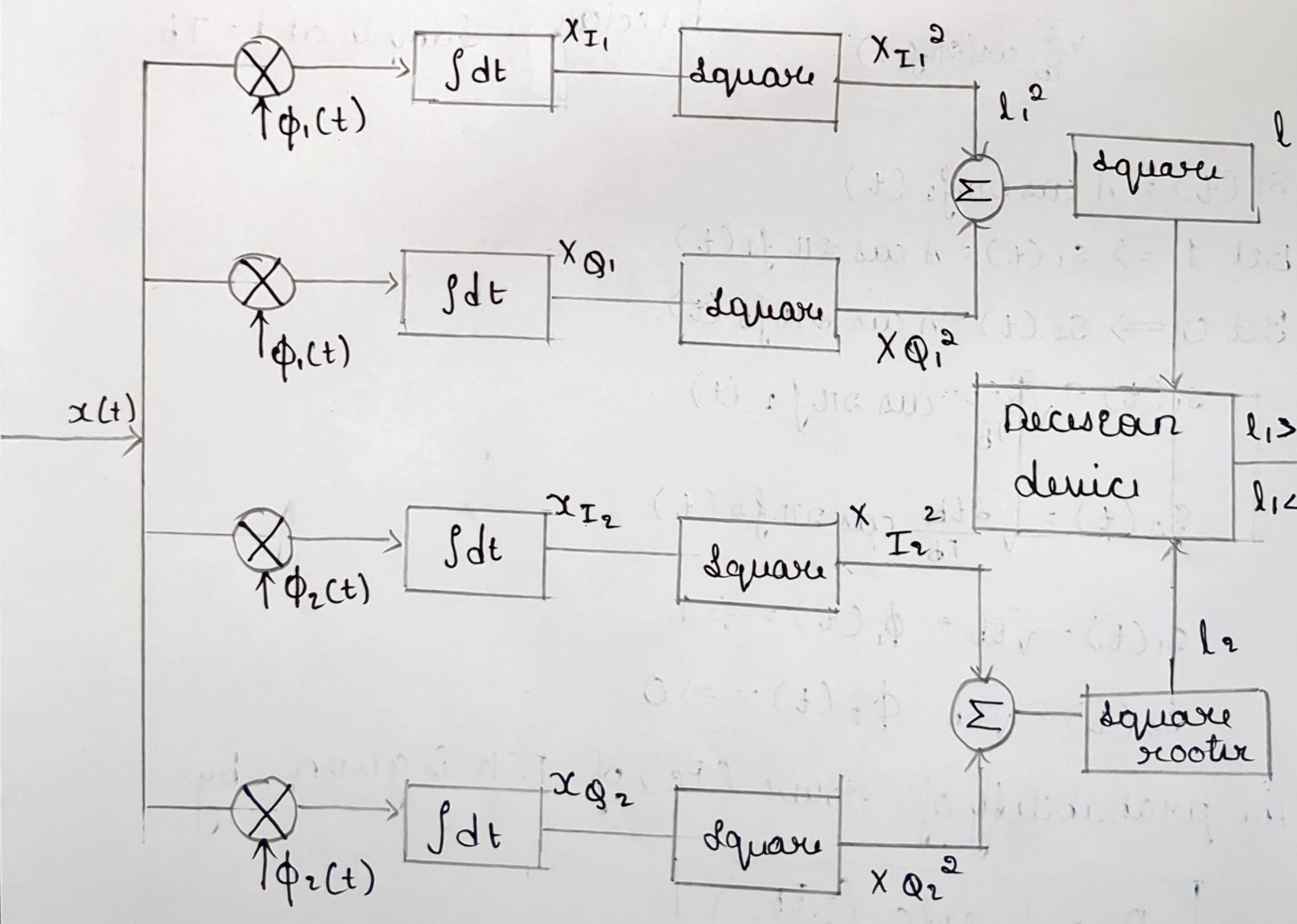
- If symbol 1 is transmitted $\boxed{x(t) = s_i(t+\theta) + w(t)}$
- If symbol 0 is transmitted $\boxed{x(t) = s_2(t+\theta) + w(t)}$
- θ is the ~~kno~~ unknown phase shift.
- $w(t)$ is white noise with zero mean and
- Power spectral density $\frac{N_0}{2}$
- There are 2 types of non-~~co~~ coherent Receivers:
  1) Matched filter
  2) Quadrature receiver.

① Matched Filter

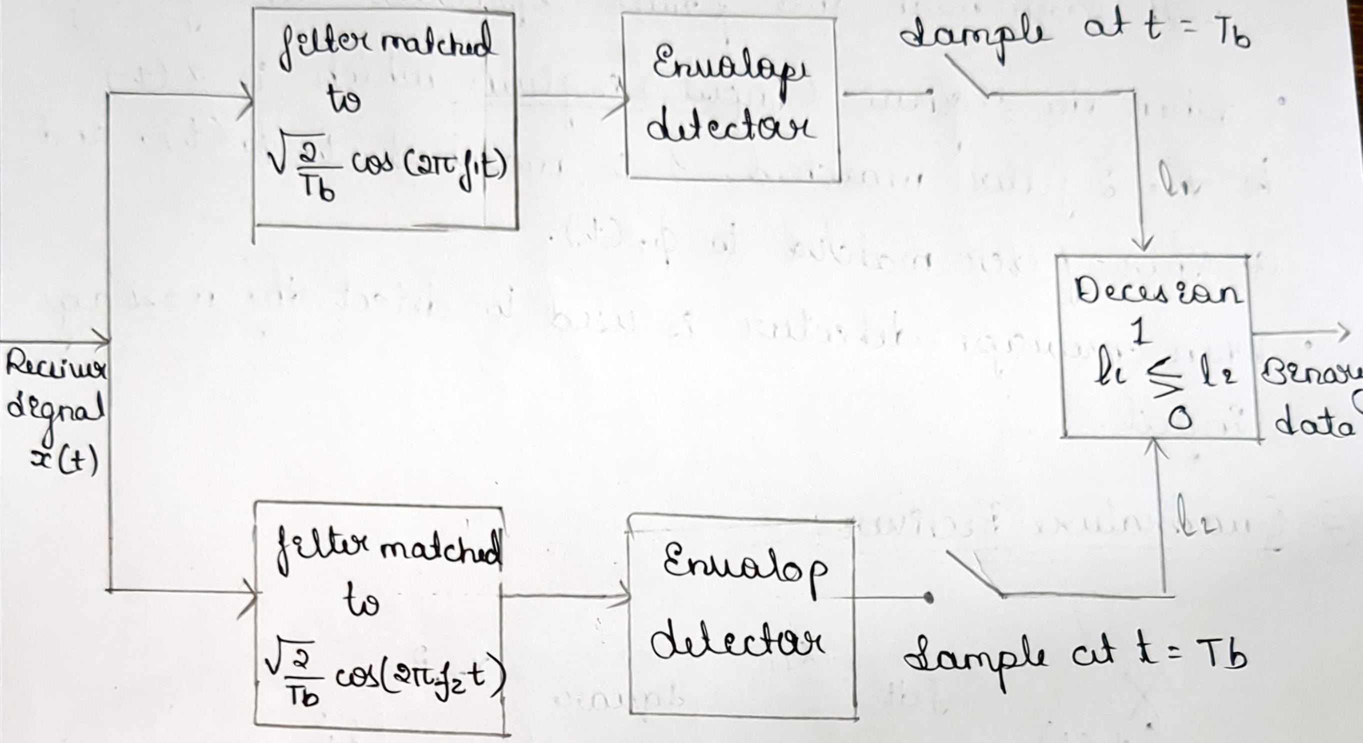


- 'θ' is the unknown phase shift, $w(t)$ is white noise with zero mean and power spectral density $\frac{N_0}{2}$.
- When the signal (Input) is given which is $x(t)$ to the 2 filter matched, 1 is matched to $\phi_1(t)$, and another filter matched to $\phi_2(t)$.
- When envelope detector is used to detect the message input.

② <u>Quadrature Receiver</u>:



★ Non-coherent Binary FSK Receiver :



$s_i(t) = A\cos 2\pi f_i(t)$

Bit 1 $\Rightarrow s_1(t) = A\cos 2\pi f_1(t)$

Bit 0 $\Rightarrow s_2(t) = A\cos 2\pi f_2(t)$

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_1(t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_2(t)$$

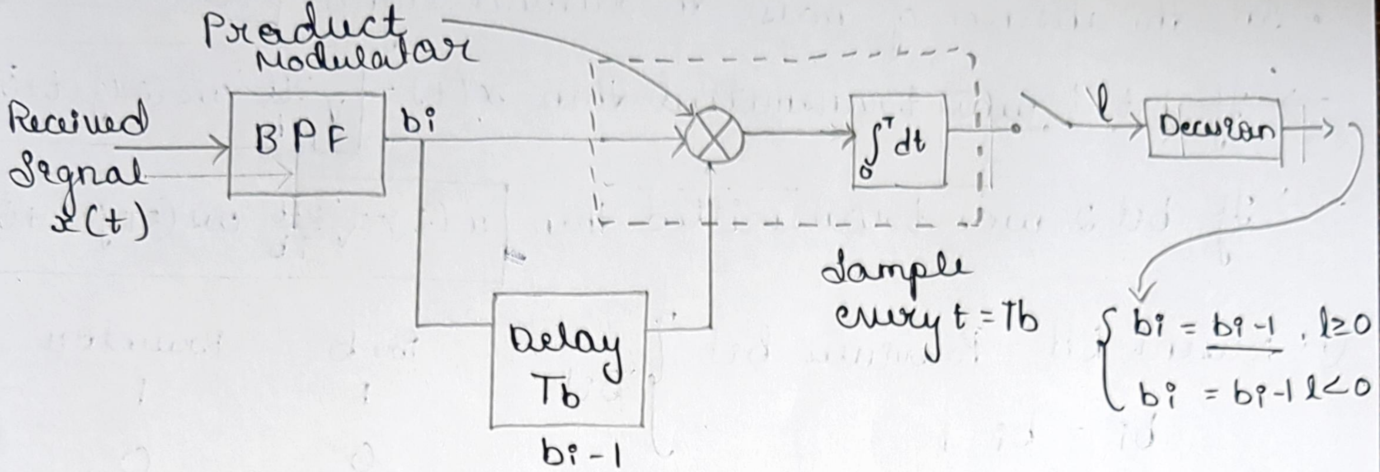
$$s_1(t) = \sqrt{E_b}\cdot\phi_1(t) \Rightarrow 1$$

$$s_2(t) = \sqrt{E_b}\,\phi_2(t) \Rightarrow 0$$

The probability of error (Pe) of FSK is given by

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\frac{-E_b}{2N_0}\right)}$$

★ Nan - coherent binary PSK receiver



- PSK wave can be given as $\boxed{s_i(t) = A\cos(2\pi f_c t + \phi)}$ so when

Bit 1 $\Rightarrow \phi = 0$; $s_1(t) = A\cos 2\pi f_c t$

Bit 0 $\Rightarrow \phi = \pi$; $s_2(t) = -A\cos 2\pi f_c t$

$$\boxed{x(t) = s_i(t) + w(t)}$$

- $x(t)$ is passed through Band pass filter is remove out of frequency signal

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$s_1(t) = \sqrt{E_b}\, \phi_1(t)$$

$$s_2(t) = \sqrt{E_b}\, \phi_1(t)$$

- There are 2 condition:

-> To detect the present bit we need to know the previous bit

-> The delay T_b will store the previous bit.

- Transmitted is same as PSK.
- In the absence of noise the received signal $\boxed{x(t) = s_i(t) + \theta}$

→ If bit 1 was transmitted then $x(t) = \boxed{\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta)}$

→ If bit 2 was transmitted then $\boxed{x(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta)}$

(i) Present bit = Previous bit $\left.\begin{matrix} \\ b_i = b_{i-1} \end{matrix}\right\}$ Bit

Pre B	Present Bit
1	1
0	0

$y(t) = x(t)^* \, x(t - T_b)$ } signal

Assumed bit 1 is transmitted

$$y(t) = \left[\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta)\right] + \left[\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta)\right]$$

$$y(t) = \frac{2E_b}{T_b} \cos^2(2\pi f_c t + \theta)$$

$$\boxed{y(t) = \frac{E_b}{T_b}\left[1 + \cos(4\pi f_c t + \theta)\right]}$$

$y(t)$ = is passed through the integrator

The out of the integrator is y

$$y = \int_0^{T_b} \frac{E_b}{T_b}\left[1 + \cos(2\pi f_c t + \theta)\right] dt$$

$$y = \frac{E_b}{T_b}\left[\int_0^T dt + \int_0^T \cos(4\pi f_c t + \theta)\, dt\right]$$

$$y = \frac{E_b}{T_b} T_b + \left[\cancel{\frac{\sin(2\pi f_c t + \theta}{2\pi f_c t + \theta}}\right]_0^{T_b}$$

0

||ly $\boxed{Y = E_b}$

||ly $\boxed{Y = E_b}$ $\quad b_i \neq b_{i-1}$

Assume bit 1 was transmitted

$$Y(t) = \left[\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t + \theta\right] + \left[\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t + 0\right]$$

After $\int_0^{T_b}$

$$\boxed{Y = -E_b}$$

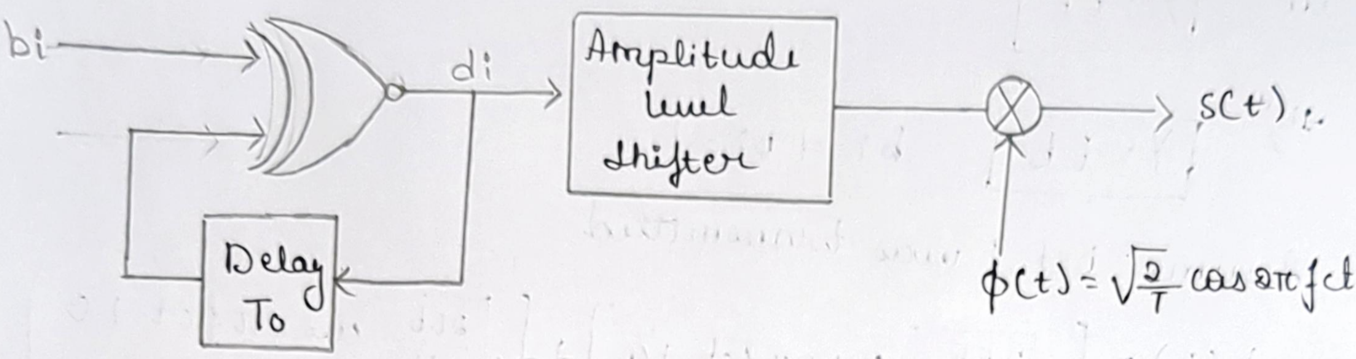
- The output of the y is sampled at $T = T_b$
- The sampled bit l is passed to decision device if $l > 0$ bit 1 was transmitted, If $l < 0$ bit 0 was transmitted

$\boxed{b_i = b_{i-1}, \ l \geq 0} \longrightarrow 1 \qquad \boxed{b_i = b_{i-1}, \ l < 0} \longrightarrow 0$

In the presence of noise if the bit is not decoded properly meaning if there is error in decoding of the bit b_i this leads to the error in consequence bits.

To overcome the error propagation we encode the data diffraction and then apply this to the BPSK system.

Differential encoding + BPSK = <u>DPSK</u>



DPSK

Differential encoded output

$d_i = b_i \oplus d_{i-1}$

$d_i = b_i \cdot d_{i-1} + \overline{b_i}\ \overline{d_{i-1}}$ —— (1)

If $b_i = d_{i-1}$; $d_i = 1$

$b_i \neq d_{i-1}$; $d_i = 0$

$b_i = \{b_0, b_1, ------, b_k\}$

$d_0 = b_0\, d_{-1} + \overline{b_0}\ \overline{d_{-1}}$

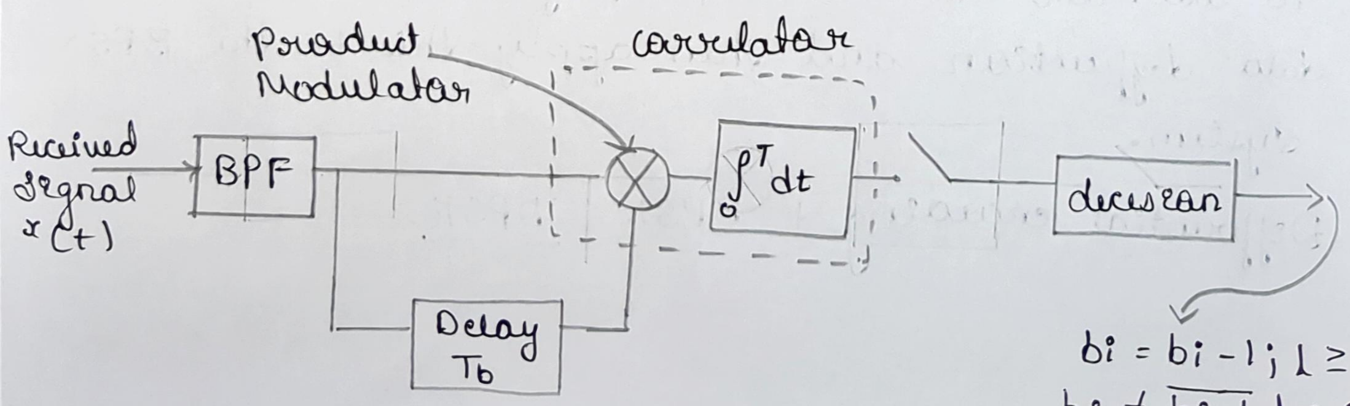
$d_1 = b_1\, d_0 + \overline{b_1}\ \overline{d_0}$

$d_2 = b_2\, d_1 + \overline{b_2}\ \overline{d_1}$

$\vdots$

$d_k = b_k\, d_{k-1} + \overline{b_k}\ \overline{d_{k-1}}$

★ **DPSK Receiver :**



$b_i = b_{i-1}$; $l \geq 1$

$b_i \neq \overline{b_{i-1}}$ $l \leq 0$

In the absence of noise the received signal is same as input signal with same phase shift

- The received signal is passed through BPF, it removes out off band frequency
- The input of the multiplier is the output of the BPF & delay T_b. (Delay T_b stores the previous output)
- Then it is XNORed and given as $Y(t)$ to the integrator
- In the presence of noise the decision device may mis interprent bit 1 as bit 0.

★ M-Ary PSK :

- In M-Ary the letter M indicates the number of faces
- The number of bits per symbal or bits / symbal $= \log_2 m$
- A M-Ary PSK signal is given by $s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \frac{2\pi i}{M}\right]$

The value of i varies from $\{0, 1, 2 \ldots M-1\}$ $\longrightarrow$ (1)

- The above equation can be written in the form of

$\cos A \cos B - \sin A \sin B$

$$s_i(t) = \sqrt{\frac{2E}{T}}\left[\cos\frac{2\pi i}{M} - \sin 2\pi f_c t \cdot \sin\frac{2\pi i}{M}\right]$$

So

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$

$$S_i(t) = \left[\sqrt{E}\,\frac{\cos 2\pi i}{M}\,\phi_1(t) - \sqrt{E}\,\sin\frac{2\pi i}{M}\,\phi_2(t)\right]$$

Orthonormal basic function.

$\frac{\pi i}{M} - \sqrt{E} \sin\frac{2\pi i}{M}$]

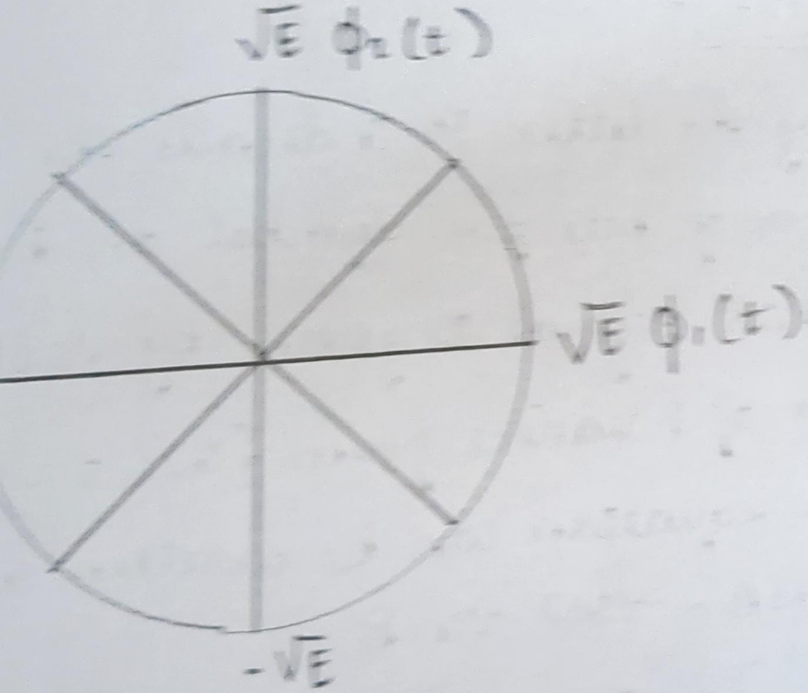
varies from $\{0, 1, 2 --- M-1\}$

shift keying method

ts / symbol $= \log_2 m = 3$ there are 8 possible

mean each symbol represent 3 bit

points are equally spaces of the circle



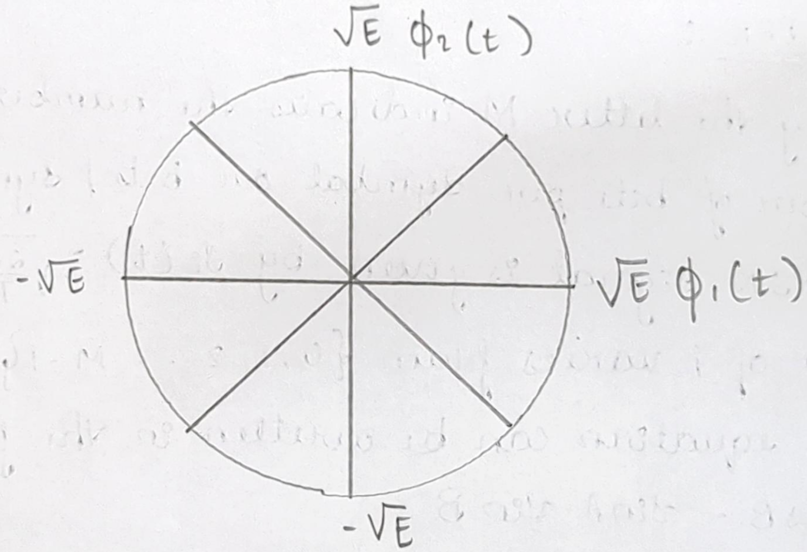
$s_i = \left[\sqrt{E} \cos\frac{2\pi i}{M} \quad - \sqrt{E} \sin\frac{2\pi i}{M} \right]$

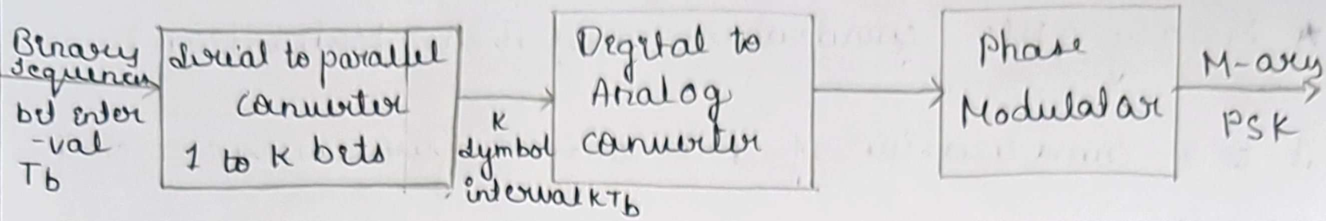
~~The value of i varies from {0, 1, 2 --- M-1}~~

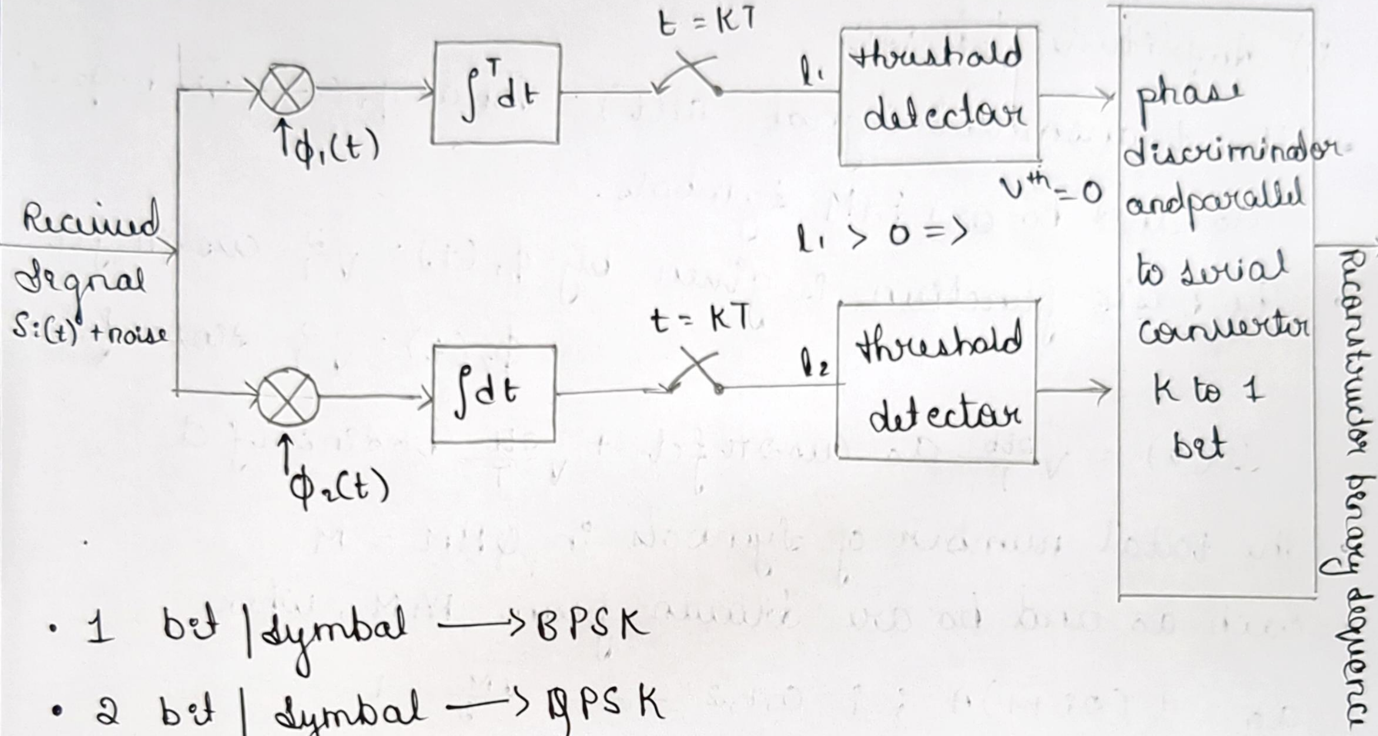
- It is the phase shift keying method
- When $M = 8$

The no. of bits / symbol $= \log_2 m = 3$ there are 8 possible faces which mean each symbol represent 3 bits

- The message points are equally spaces of the circle.







- 1 bit | symbol $\longrightarrow$ BPSK
- 2 bit | symbol $\longrightarrow$ QPSK
- M bit | symbol $\longrightarrow$ M-ary PSK
- In the presence of noise the decision device may interpret the bit 0 as 1 and 1 as 0, causes the intersymbol interference hence the above Pe equation is used

$$\boxed{P_e = \text{erfc}\left[\sqrt{\frac{E}{N_0}}\,\sin\left(\frac{\pi}{m}\right)\right]}$$

★ M-ary QAM (Quadrature Amplitude Modulation)

- It is a combination of PAM signal Quadrature components
  i) I and Q $\Rightarrow$ PAM
  ii) Amplitude / phase
- The transmitted signal $s_i(t) = \sqrt{E_b}\, a_0\, \phi_1(t) + \sqrt{E_b}\, b_0 \phi_2(t)$
  a_0 and b_0 are PAM symbols.
- The basis function is given by $\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$
  $\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$$s_i(t) = \sqrt{\frac{2E_b}{T}}\, a_0 \cos 2\pi f_c t + \sqrt{\frac{2E_b}{T}}\, b_0 \sin 2\pi f_c t$$

- The total number of symbols in QAM = M
- Each a_0 and b_0 are drawn from PAM, where
  $a_0 = \pm(2i+1)A$; $i = 0, 1, 2 \dots \frac{\sqrt{M}}{2} - 1$
  $b_0 = \pm(2j+1)A$; $j = 0, 1, 2 \dots \frac{\sqrt{M}}{2} - 1$
- The vector representation of the signal $s_i = (a_0\sqrt{E_b}, b_0\sqrt{E_b})$
  a_0 and b_0 can take l values which is given by
  $\{(-L+1), (-L+2), (-L+3) \dots (L-2), (L-1)\}$

Here we are using 16 QAM

$$\boxed{M = 16} \Rightarrow L^2 = M$$

$$\boxed{L = 4}$$

$\{(-L+1), (-L+3), (L-3), (L-4)\}$

$\{\{-3, -1, 1, 3\}$

$$a_0 b_0 = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix}$$

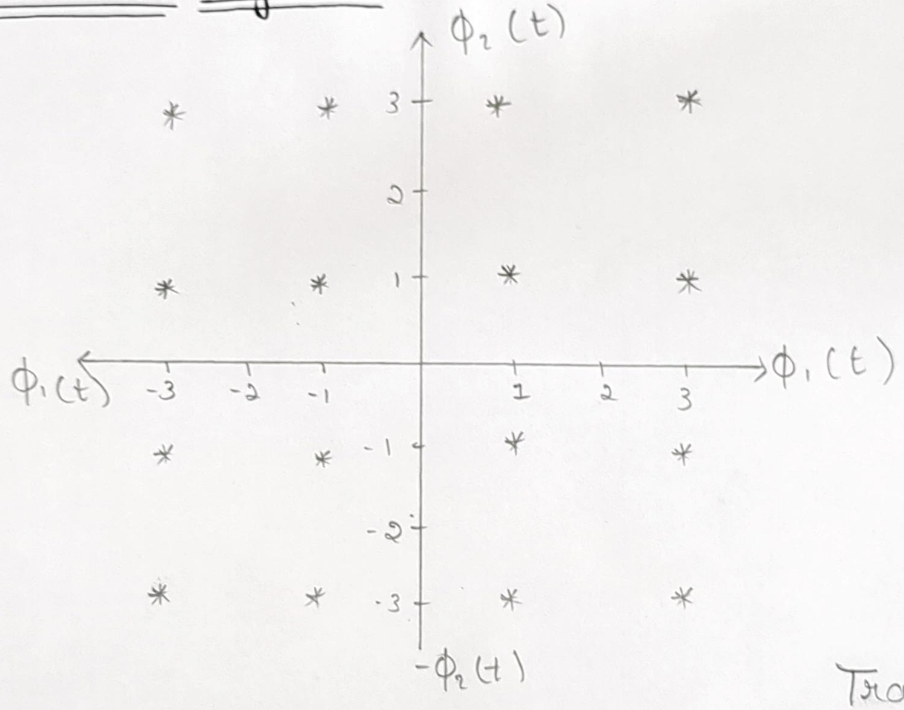
Binary	PAM
00	-3
01	-1
10	1
11	3

Binary digital corresponds to QAM Symbal

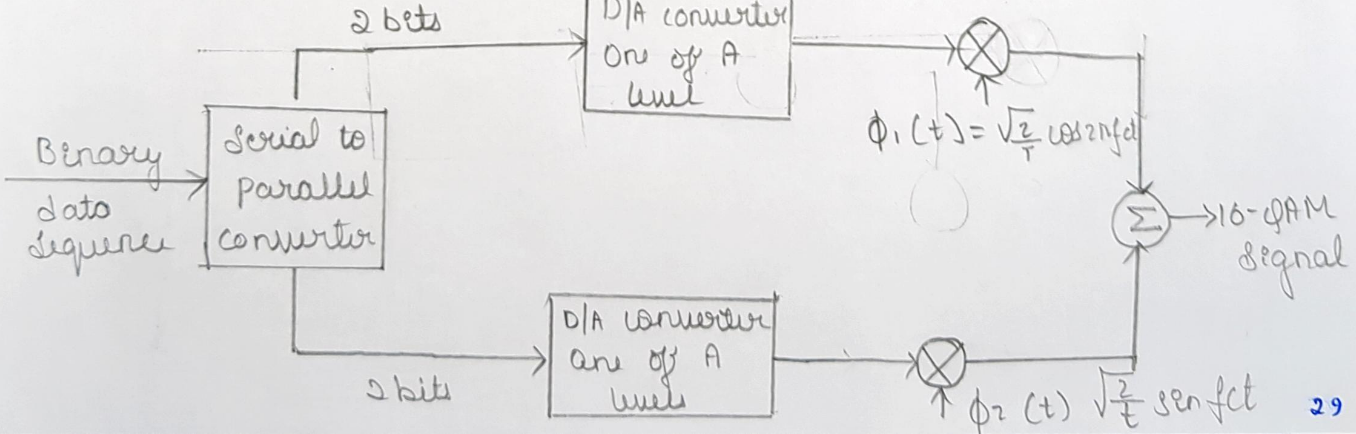
$1011 \Longrightarrow (1,3)$

$0001 \Longrightarrow (3,-1)$.

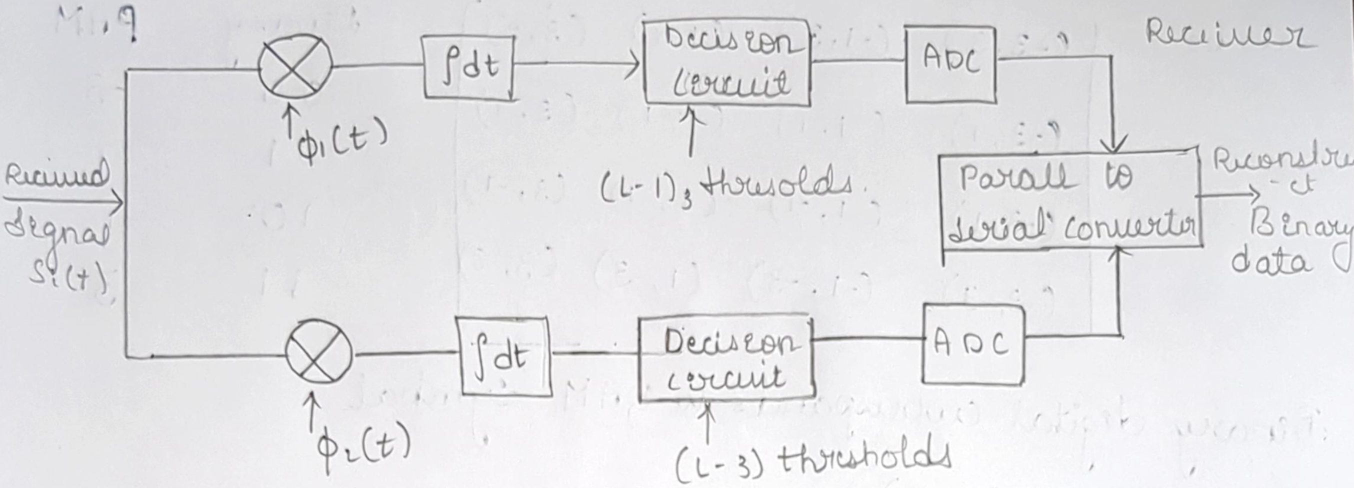
★ **Consillation diagram:**



Transmitter



M.Q Reciever



- The Pe of a QAM $= 2\left[1 - \frac{1}{\sqrt{m}}\right] \text{erfc}\left[\sqrt{\frac{3E_b}{2(M-1)N_0}}\right]$

Problems

① A binary FSK system transmitts data at a rate of two (2) Mega bits/second over an AWGN channel. the noise power spectral density $\frac{N_0}{2} = 10^{-20}$ W/Hz. Determine The probability of error for a coherent FSK. Assume the amplitude of the received signal as 1 microvolt

Given erfc (228) = 0.99959.

$$\boxed{R_B = 2M \text{ bps}}$$

$$T_b = \frac{1}{R_b} = \frac{1}{2M\text{bps}} = \cancel{50}\,0.5\,\mu S$$

$$\Rightarrow \frac{N_0}{2} = 10^{-20} \text{ W/Hz} \Rightarrow N_0 = 2 \times 10^{-20} \text{ W/Hz}$$

$$A = 1 \mu V$$

$$\boxed{P_e = \frac{1}{2} \text{erfc} \sqrt{\frac{E_b}{2N_0}}}$$

$$E_b = PT_b$$

$$P = \frac{V_{rms}^2}{R}$$

$$P = \frac{V^2}{2R}$$

Assume R as 1 ohm.

$$P = \frac{(1 \times 10^{-6})^2}{2}$$

$$\boxed{P = 5 \times 10^{-13}}$$

$$E_b = (5 \times 10^{-13})(\cancel{50}\,0.5 \times 10^{-7})$$

$$E_b = 2.5 \times 10^{-19}$$

$$Pe = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{2.5 \times 10^{-19}}{2 \times (2 \times 10^{-20})}}$$

$$Pe = \frac{1}{2} \operatorname{erfc}\left(\frac{5}{2}\right)$$

$$Pe = \frac{1}{2}[1 - \operatorname{erfc}(2.5)] \quad Pe = 0.905 \times 10^{-3}$$

(2) In a binary PSK system binary 1 is represented by $s_1(t) = A \cos w_c t$ and binary 0 is represented by $s_2(t) = A \cos w_c t$ and transmitted by AWGN channel at a rate of 0.2 ms. The carrier amplitude at the receiver is 1mV. the power spectral density $\frac{N_0}{2} = 10^{-12}$ W/Hz find the average probability of error for coherent B PSK system.

$$\operatorname{erfc}(x) = 1 - \operatorname{erfc}(x)$$

$$S_1(t) = A \cos W_c t$$

$$S_2(t) = A \cos W_c t$$

$$R_b = 0.2 \text{ msec}$$

$$A = 1 \times 10^{-3} \, V$$

$$\frac{N_0}{2} = 10^{-11} \text{ W/Hz}$$

$$T_b = \frac{1}{R_b} = \frac{1}{0.2 \times 10^{-3}} = \boxed{5 \times 10^{3} \text{ sec}}$$

$$E_b = P T_b \Rightarrow P = \frac{V^2_{rms}}{R}$$

$$P = \frac{V^2}{2R}$$

$$R = 1\,\Omega$$

$$P = \frac{(1 \times 10^{-3})^2}{2}$$

$$E_b = (5 \times 10^{-7})(5 \times 10^{3})$$

$$E_b = 2.5 \times 10^{-3}$$

$$\boxed{P = 5 \times 10^{-7}}$$

$$Pe = \frac{1}{2} \text{erf} \sqrt{\frac{Eb}{2No}}$$

(3) The binary data is transmitted over AWGN channel using BPSK at a rate of 1M bits/sec. It is desired to have an average probability of error $Pe \leq 10^{-4}$. The noise power spectrum density $\frac{No}{2} = 10^{-12}$ W/Hz. Determine the average carrier power at the receiver input if the detector is of coherent type assume the value of error function to be erfc $(3.5) = 0.0025$

$Rb = 1 Mbps$

$Pe \leq 10^{-4}$

$$\frac{No}{2} = 10^{-12} \, W/Hz$$

$$Pe = \frac{1}{2} \text{erfc} \sqrt{\frac{Eb}{No}}$$

$$10^{-4} = \frac{1}{2} \text{erfc} \sqrt{\frac{Eb}{No}}$$

$Eb = P \cdot Tb$

$$\text{erfc} \sqrt{\frac{Eb}{No}} = 2 \times 10^{-4}$$

$$\text{erfc}(u) = 1 - \text{erfc}(u)$$

$$= 1 - 2 \times 10^{-4}$$

$$= 0.9998$$

from error function table $u \simeq 2.8$ or 2.9

$$\sqrt{\frac{Eb}{No}} \; 2.8 \Longrightarrow \frac{Eb}{No} = (2.8)^2$$

$E_b = 2 \times 10^{-12} \times 7.84$

$\cancel{P_{Tb}}$ $P_{Tb} = 1.568 \times 10^{-11}$

$P = \frac{1.568 \times 10^{-11}}{T_b} = 15.6\ \mu W$

$\boxed{P = 15.6\ \mu W}$