Module 1 Notes for

# “DIGITAL COMMUNICATION”

# [BEC503]

**Prepared by:**

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# AKSHAYA INSTITUTE OF TECHNOLOGY

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## DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

## VISION

To produce competent engineering professionals in the field of Electronics and Communication Engineering by imparting value based quality technical education to meet the societal needs and to develop socially responsible citizens.

## MISSION

**M1:** To provide strong fundamentals and technical skills in the field of Electronics and Communication Engineering through effective teaching learning process.

**M2:** Enhancing employability of the students by providing skills in the fields of VLSI, Embedded systems, Signal processing, etc., through Centre of Excellence.

**M3:** Encourage the students to participate in co-curricular and extra-curricular activities that creates a spirit of social responsibility and leadership qualities.

## Program Specific Outcomes (PSOs)

*After Successful Completion of Electronics and Communication Engineering Program Students will be able to*

1. Apply fundamental knowledge of core. Electronics and Communication Engineering in the analysis, design and development of Electronics Systems as well as to interpret and synthesize experimental data leading to valid conclusions.
2. Exhibit the skills gathered to analyze, design, develop software applications and hardware products in the field of embedded systems and allied areas.

## Program Educational Objectives (PEOs)

**PEO1:** Graduates exhibit their innovative ideas and management skills to meet the day to day technical challenges.

**PEO2:** Graduates utilize their knowledge and skills for the development of optimal solutions to the problems in the field of Electronics and Communication Engineering..

**PEO3:** Graduates exhibit good interpersonal skills, leadership qualities and adapt themselves for life-long Learning

| DIGITAL COMMUNICATION | | Semester | 5 |
|---|---|---|---|
| Course Code | BEC503 | CIE Marks | 50 |
| Teaching Hours/Week (L:T:P: S) | 4:0:0:0 | SEE Marks | 50 |
| Total Hours of Pedagogy | 50 Hours | Total Marks | 100 |
| Credits | 04 | Exam Hours | 3 Hours |
| Examination type (SEE) | Theory | | |

**Course objectives:**

- Understand the concept of signal processing of digital data and signal conversion to symbols at the transmitter and receiver.
- Compute performance metrics and parameters for symbol processing and recovery in ideal and corrupted channel conditions.
- Understand the principles of spread spectrum communications.
- Understand the basic principles of information theory and various source coding techniques.
- Build a comprehensive knowledge about various Source and Channel Coding techniques.
- Discuss the different types of errors and error detection and controlling codes used in the communication channel.
- Understand the concepts of convolution codes and analyze the code words using time domain and transform domain approach.

**Teaching-Learning Process (General Instructions)**

These are sample Strategies, which teachers can use to accelerate the attainment of the various course outcomes.

1. Lecture method (L) does not mean only the traditional lecture method, but a different type of teaching method may be adopted to develop the outcomes.
2. Arrange visits to nearby PSUs such as BHEL, BEL, ISRO, etc., and small-scale communication industries.
3. Show Video/animation films to explain the functioning of various modulation techniques, Channel, and source coding.
4. Encourage collaborative (Group) Learning in the class
5. Ask at least three HOTS (Higher-order Thinking) questions in the class, which promotes critical thinking
6. Adopt Problem Based Learning (PBL), which fosters students' Analytical skills, develop thinking skills such as the ability to evaluate, generalize & analyze information rather than simply recall it.
7. Topics will be introduced in multiple representations.
8. Show the different ways to solve the same problem and encourage the students to come up with their own creative ways to solve them.
9. Discuss how every concept can be applied to the real world - and when that's possible, it helps improve the students' understanding.

**Module-1**

**Bandpass Signals to Equivalent Lowpass:** Hilbert Transform, Pre-envelopes, Complex envelopes of Band-pass Signals, Canonical Representation of Bandpass signals.

**Signalling over AWGN Channels-** Introduction, Geometric representation of signals, Gram- Schmidt Orthogonalization procedure, Conversion of the continuous AWGN channel into a vector channel , Optimum receivers using coherent detection: ML Decoding, Correlation receiver, matched filter receiver.

**Module-2**

**Digital Modulation Techniques:** Phase shift Keying techniques using coherent detection: generation, detection and error probabilities of BPSK and QPSK, M–ary PSK, M–ary QAM. Frequency shift keying techniques using Coherent detection: BFSK generation, detection and error probability. BFSK using Noncoherent Detection, Differential Phase Shift Keying.

**Module-3**

| |
|---|
| **Information theory:** Introduction, Entropy, Source Coding Theorem, Lossless Data Compression Algorithms, Discrete Memoryless Channels, Mutual Information, Channel capacity, Channel Coding Theorem, Information Capacity Law (Statement). |
| **Module-4** |
| **Error Control Coding:** Error Control Using Forward error Correction,<br>**Linear Block Codes:** Definitions, Matrix Descriptions, Syndrome and its properties, Minimum distance Considerations, Syndrome Decoding, Hamming Codes.<br>**Cyclic Codes:** Properties, Generator and Parity Check Polynomial and matrices, Encoding, Syndrome computation, Examples. |
| **Module-5** |
| **Convolutional Codes:** Convolutional Encoder, Code tree, Trellis Graph and State graph, Recursive systematic Convolutional codes, Optimum decoding of Convolutional codes, Maximum Likelihood Decoding of Convolutional codes: The Viterbi Algorithm, Examples. |

**Course outcome (Course Skill Set)**

At the end of the course, the student will be able to :

1. Apply the concept of signal conversion to vectors in communication transmission and receiption.
2. Perform the mathematical analysis of digital communication systems for different modulation techniques.
3. Apply the Source coding and Channel coding principles for the discrete memoryless channels.
4. Compute the codewords for the error correction and detection of a digital data using Linear Block Code, Cyclic Codes and Convolution Codes.
5. Design encoding and decoding circuits for Linear Block Code, Cyclic Codes and Convolution Codes.

AKSHAYA INSTITUTE OF TECHNOLOGY

Department of Electronics & Communication Engineering.

Subject: Digital communication [BEC503]

Sem : $5^{th}$ semester

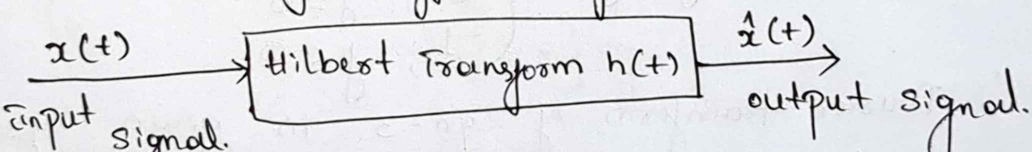
By. Sindhu. R
Assistant Professor

# Module - 1

## Bandpass Signals to Equivalent lowpass.

1. **Hilbert Transform:** It is a method of separating signals based on Phase selectivity which uses phase shift b/n the signals..

* when the phase angles of all the components of a given signal are shifted by $\pm 90^\circ$, the resulting function of time is known as Hilbert transform of the signal.
* It is also called as Quadrature filter as it provides $\pm 90^\circ$ phase shift.
* It is a $\pm 90^\circ$ phase shifter in time domain without changing Amplitude value.
* It is not an equivalent representation of the signal but it is entirely a different signal in the same domain.




$$\hat{x}(t) = x(t) * h(t).$$

i.e,
$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{(t-\tau)} \, d\tau \quad \text{--- (1)}$$

Hilbert transform is a linear function so its inverse hilbert transform is given by.

$$x(t) = \frac{-1}{\pi} \int_{-\infty}^{\infty} \frac{\hat{x}(\tau)}{(t-\tau)} d\tau \quad — ②$$

→ From defintion Hilbert transform $\hat{x}(t)$ is the convolution of $x(t)$ with the time function $h(t) = \frac{1}{\pi t}$

$$\therefore \hat{x}(t) = x(t) * \frac{1}{\pi t} \longrightarrow ③$$

WKT Convolution in time domain is equal to product in Frequency domain

Applying F.T to equation ③

$$FT\{x(t)\} = X(f)$$

$$FT\{\hat{x}(t)\} = \hat{x}(f)$$

$$FT\left\{\frac{1}{\pi t}\right\} = -j\,sgn(f)$$

WKT

$$sgn(t) \xleftrightarrow{FT} \frac{2}{j\omega} = \frac{1}{j\pi f} \quad [\text{Note} \to \omega = 2\pi f]$$

where $sgn(f)$ is the signum function, defined in frequency domain as

$$sgn(f) = \begin{cases} 1 ; & f > 0 \\ 0 ; & f = 0 \\ -1 ; & f < 0 \end{cases}$$

∴ Fourier transform of eqn-3 is given by.

$$\hat{x}(f) = -j\,sgn(f) \cdot X(f) \longrightarrow ④$$

Above equation-4 states that the given fourier transformable Signal $x(t)$.

we will get Hilber transform $\hat{x}(t)$ by passing $x(t)$ through. LTI system whose frequency response are equal to $-j\,sgn(f)$

If we multiply the spectrum of $x(t)$ by $-j$ & $-ve$ spectrum ~~by~~ of $x(t)$ by $+j$. we obtain the Hilbert transform of $x(t)$ in Frequency domain.

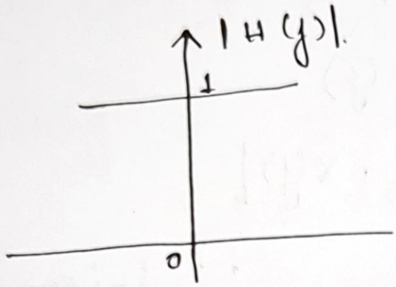



Fig 1.1 - Magnitude response.

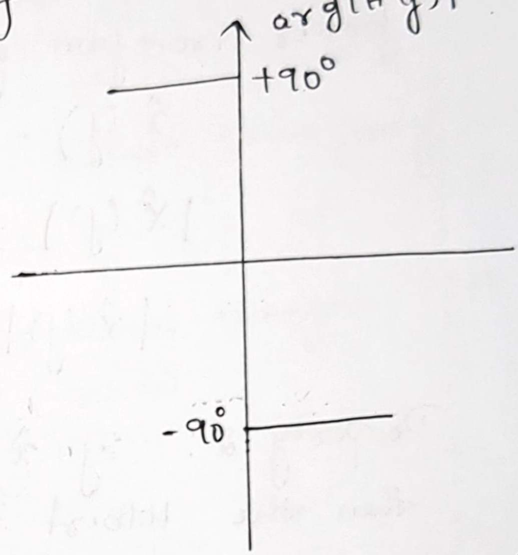
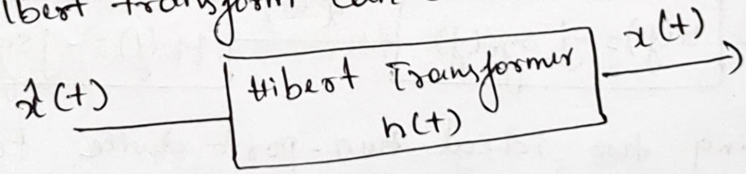



Fig 1.2 → phase response of Hilbert Transform.

Inverse Hilbert transform can be written as.




$$x(f) = \hat{x}(f) * h(f)$$

$$x(f) = \hat{x}(f) \cdot j\,\mathrm{sgn}(f)$$

## 2. Properties of Hilbert Transform!

1 Property 1 : A signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ have the same magnitude spectrum

$$|X(f)| = |\hat{X}(f)|$$

Proof: The signal $x(t)$ be fourier transformable with fourier transform $X(f)$

let $\hat{x}(t)$ be its hilbert transform & $\hat{X}(f)$ be the fourier transform of $\hat{x}(t)$ then.

$$\hat{X}(f) = -j\,\mathrm{sgn}(f) \cdot X(f)$$

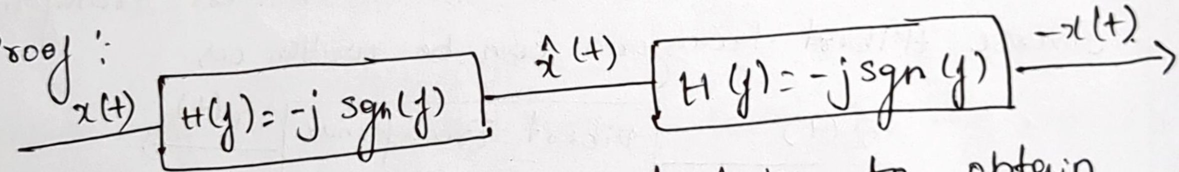
$$|\hat{X}(f)| = |-j\,\mathrm{sgn}(f) \cdot X(f)|$$

$$|\hat{X}(f)| = |X(f)| \qquad \because |-j\,\mathrm{sgn}(f)| = 1$$

Property 2: If $\hat{x}(t)$ is the Hilbert transform of $x(t)$ then the Hilbert transform of $\hat{x}(t)$ is $-x(t)$

$$\arg[X(f)] = -\arg\{\hat{X}(f)\}$$

Proof:




cascading two ideal two-port device to obtain double Hilbert Transform.

$$\hat{x}(t) = x(t) * h(t).$$

Taking FT of above equation.

$$\hat{X}(f) = X(f) \cdot H(f)$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f) \cdot X(f) \quad \text{—— (1)}$$

Hilbert transform of $\hat{x}(t)$ is given by $\hat{\hat{x}}(t)$.

i.e $\hat{\hat{x}}(t) = \hat{x}(t) * h(t)$

Taking fourier transform we get

$$\hat{\hat{x}}(f) = \hat{x}(f) \cdot H(f).$$

$$\hat{\hat{x}}(f) = [-j\,\text{sgn}(f) \cdot x(f)]\,[-j\,\text{sgn}(f)]$$

WKT = $\text{sgn}(f) \cdot \text{sgn}(f) = 1$ & $j^2 = -1$

$$\therefore \quad \hat{\hat{x}}(f) = -x(f) \quad \text{— ②}$$

Taking IFT of eq (2). we get Hilbert transform of $\hat{x}(t)$

i.e $\quad \hat{\hat{x}}(t) = -x(t) \quad$ — ③

Property 3: A signal $x(t)$ & its Hilbert transform $\hat{x}(t)$ are orthogonal over time interval $(-\infty, \infty)$

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = 0.$$

Proof: To prove that $x(t)$ & $\hat{x}(t)$ are orthogonal we have to prove that

i.e $\quad \int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = 0.$

using Parseval's theorem $E_{TD} = E_{FD}$

i.e $\quad \int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = \int_{-\infty}^{\infty} x(f) \cdot \hat{x}^*(f)\,df \quad$ — ①

WKT for a real valued signal.

$$\hat{x}^*(f) = \hat{x}(-f).$$

Rewriting eq ① we get

$$\int_{-\infty}^{\infty} x(t) \cdot \hat{x}(t)\,dt = \int_{-\infty}^{\infty} x(f) \cdot \hat{x}(-f)\,df. \quad \text{— ②}$$

WKT, $\hat{x}(f) = -j\,\mathrm{sgn}(f)\cdot x(f)$

$\hat{x}(-f) = -j\,\mathrm{sgn}(-f)\cdot x(-f) = j\,\mathrm{sgn}(f)\cdot x(-f).$

Substituting the value of $\hat{x}(-f)$ in eq-② we get

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} x(f)\cdot j\,\mathrm{sgn}(f)\,x(-f)\,df$$

$$= j\int_{-\infty}^{\infty} x(f)\cdot x(-f)\,\mathrm{sgn}(f)\,df.$$

$$= j\int_{-\infty}^{\infty} |x(f)|^2\cdot \mathrm{sgn}(f)\,df.$$

$$\therefore \int_{-\infty}^{\infty} x(t)\cdot\hat{x}(t)\,dt = 0$$

WKT, $x(f)\cdot x(-f) = |x(f)|^2$ which is even function & $\mathrm{sgn}(f)$ is odd function. Hence integration of odd function over the limits $-\infty$ to $\infty$ is zero.

## Hilbert transform of low pass signal.

The below given figure shows fourier transform of a lowpass signal $x(t)$ whose frequency content extends from $-w$ to $+w$

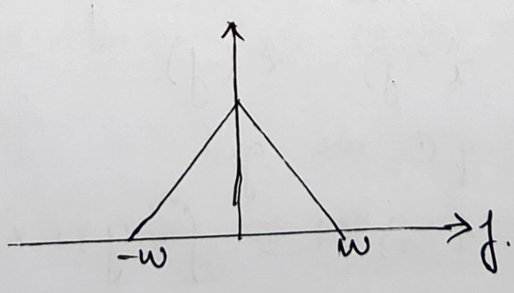



fig 1.3. Spectrum of singnal $x(t)$

Applying hilbert Transform to this signal will give a new signal $\hat{x}(t)$ whose fourier transform $\hat{x}(f)$ is shown below.

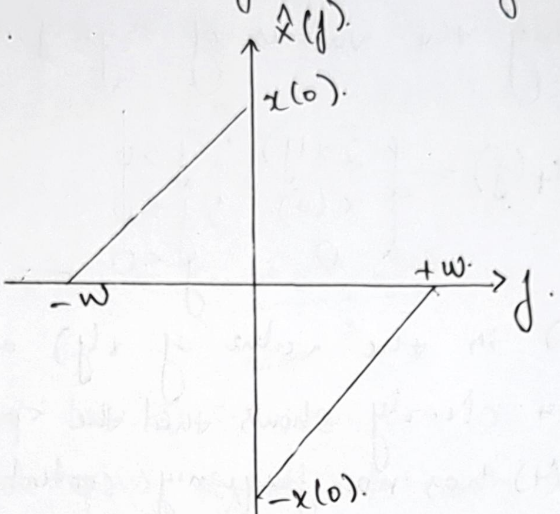



Fig 1.4 → spectrum of Hilbert transform of $\hat{x}(t)$

**Pre-Envelopes:** in certain applications we need to transmit only positive frequency or transmit only negative frequency. At that time use pre-envolope.

$$x(t) = x_+(t) + x_-(t) \quad — ①$$

A real valued signal $x(t)$ can be written in terms of its negative pre-envolope & positive pre-envelope.

The positive pre-envolope of signal $x_+(t)$ can be written as

$$x_+(t) = x(t) + j\hat{x}(t) \quad — ②$$

Applying fourier transform for eq-② we get

$$x_+(f) = x(f) + j[-j \operatorname{sgn}(f) \cdot x(f)]$$

$$= x(f) + \operatorname{sgn}(f) \cdot x(f) \qquad (\because j^2 = -1)$$

$$x_+(f) = x(f)[1 + \operatorname{sgn}(f)] \quad — ③$$

WKT

$$\operatorname{sgn}(f) = \begin{cases} 1 & ; f>0 \\ 0 & ; f=0 \\ -1 & ; f<0 \end{cases}$$

Substituting the values of sgn(f) in eqn -③

we get

$$X_+(f) = \begin{cases} 2X(f) & ; f>0 \\ X(0) & ; f=0 \\ 0 & ; f<0 \end{cases} \quad \text{—} \quad ④$$

where $X(0)$ is the value of $X(f)$ at the origin $f=0$.

the eqn-4 clearly shows that the pre-envelop of the signal $x(t)$ has no frequency content for all negative frequencies.

The negative pre envolope of the signal $x_-(t)$ can be written as

$$x_-(t) = x(t) - j\hat{x}(t) \quad \text{—} \quad ⑤$$

Applying fourier transform

$$X_-(f) = X(f) - j\hat{X}(f)$$

$$= X(f) - j[-j\operatorname{sgn}(f)X(f)]$$

$$X(f) - \operatorname{sgn}(f)X(f)$$

$$X_-(f) = X(f)[1 - \operatorname{sgn}(f)] \quad \text{—} \quad ⑥$$

Applying the values of sgn(f) to eqn ⑥ we get

$$X_-(f) = \begin{cases} 0 & ; f>0 \\ X(f) & ; f=0 \\ 2X(f) & ; f<0 \end{cases} \quad \text{—} \quad ⑦$$

From equation-⑦ it is clear that pre-envelope for negative frequencies does not contain any positive frequencies

The sum of $x_+(t)$ & $x_-(t)$ is exactly twice the original signal $x(t)$

i.e substitue eq-7 & eq equn 6 in eqn ①

$$x_+(t) + x_-(t) = [x(t) + j\hat{x}(t)] + [x(t) - j\hat{x}(t)]$$

$$x_+(t) + x_-(t) = 2x(t)$$

$$x(t) = \frac{x_+(t) + x_-(t)}{2}$$

## Pre-envelope of a low pass signal.

Consider a low pass signal $x(t)$ with its fourier transform $x(f)$ as shown below.

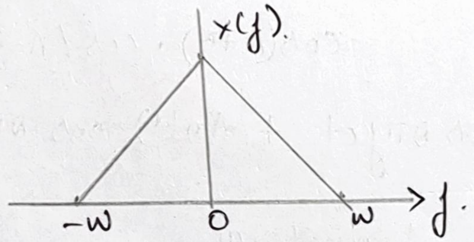



Fig: 1.5

~~Fig: 1.6~~ spectrum of pre envolope $x_+(t)$ & $x(t)$. are shown below

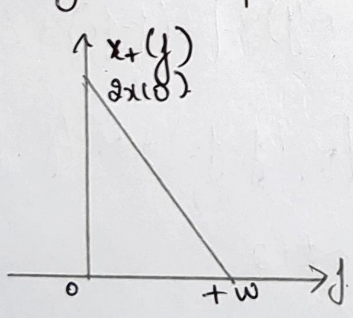



Fig: 1.6 Spectrum of $x_+(t)$
$(0 < f < W)$

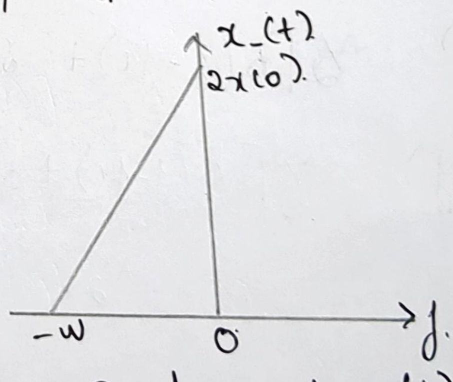



Fig: 1.7 Spectrum of $x(t)$
$(-W < f < 0)$

## Complex envelope of a Bandpass Signal:

Complex envolope is applied to Bandpass signals the only requirement is that the bandpass signal has to be fourier transformable.

Pre envolope is related to only lowpass & high pass signal.

Complex envolope is related to bandpass signal.

Bandpass signals are emplified by signals modulated on to a sinusoidal carrier.

If $s(t)$ is a amplitude modulated waven which is given by $s(t) = A/2 \cos 2\pi f_c t + A/2 \cos 2\pi (f_c + f_m)t + \frac{A}{2} \cos 2\pi (f_c + f_m)$

$$\cos(A+B) \cdot \cos(A-B)$$

$$s(t) = A/2 \cos 2\pi f_c t + A m(t) \cos 2\pi f_c t.$$

By applying fourier transform

$$S(f) = A/2 [\delta(f - f_c) + \delta(f + f_c)] + A/2 \, M(f) * [\delta(f - f_c) + \delta(f + f_c)]$$

$$S(f) = A/2 [\delta(f - f_c) + \delta(f + f_c)] + A/2 [M(f - f_c) + M(f + f_c))$$

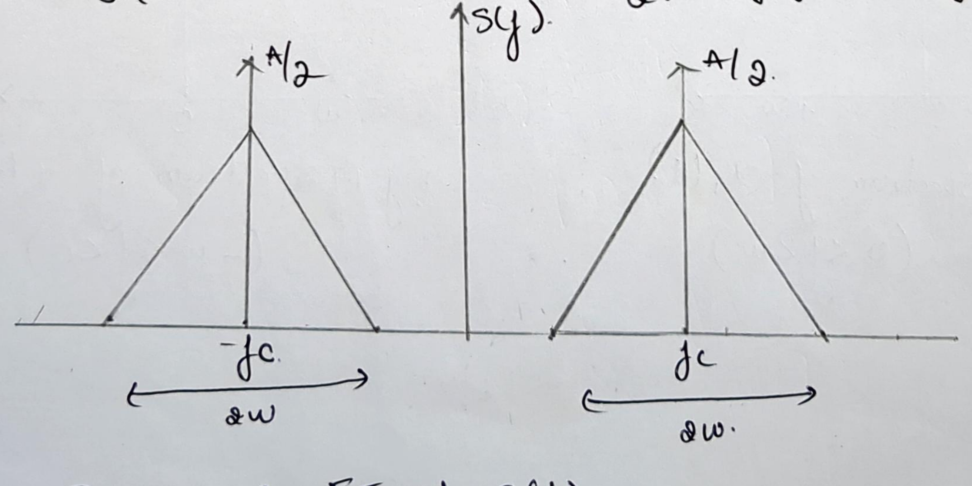



Figure: 1.8. FT of $s(t)$