VISVESVARAYA TECHNOLOGICAL UNIVERSITY

JNANA SANGAMA, BELGAVI -590 014

"PRINCIPELS OF COMMUNICATION SYSTEMS (22BEC402)"

INTEGRATED

(Theory / Practical)

(Effective from the academic Year 2024-2025)

Study Material for PCS

EVEN SEMESTER – IV

Subject Code: **22BEC402**

(Choice Based Credit System)

Prepared by:	HOD
Prof. Pradeep Kumara V H B.E., M.Tech. Asst. Professor. Dept. of E&CE, AIT, Tumkur.	**Dr. Nagendra kumar M** Ph.D.,MISTE., MIETE. Professor & HOD., Dept. of E&CE, AIT, Tumkur.

Department oF Electronics & Communication Engineering

Akshaya Institute of Technology

(Recognized by AICTE, New Delhi and Affiliated to Visvesvaraya Technological , University, Belagavi)

Akshaya Institute of Technology lingapura, Obalapura post, Koratagere Road, Tumakuru-distric-572106, Karnataka State, INDIA.

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(Recognized by AICTE, New Delhi and Affiliated to Visvesvaraya Technological , University, Belagavi)
Akshaya Institute of Technology lingapura, Obalapura post, Koratagere Road,
Tumakuru-distric-572106, Karnataka State, INDIA.

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Department oF Electronics & Communication Engineering

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EVEN SEMESTER – III

Subject Code: 22BEC402

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STUDENT'S NAME: ....

USN: ....

BRANCH: ....

SECTION: .... **YEAR:** ....

AKSHAYA INSTITUTE OF TECHNOLOGY

Lingapura, Obalapura Post, Koratagere Road, Tumakuru - 572106

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VISION

To produce competent engineering professionals in the field of Electronics and Communication Engineering by imparting value based quality technical education to meet the societal needs and to develop socially responsible citizens.

MISSION

M1: To provide strong fundamentals and technical skills in the field of Electronics and Communication Engineering through effective teaching learning process.

M2: Enhancing employability of the students by providing skills in the fields of VLSI, Embedded systems, Signal processing, etc., through Centre of Excellence.

M3: Encourage the students to participate in co-curricular and extra-curricular activities that creates a spirit of social responsibility and leadership qualities.

Program Specific Outcomes (PSOs)

After Successful Completion of Electronics and Communication Engineering Program Students will be able to

1. Apply fundamental knowledge of core. Electronics and Communication Engineering in the analysis, design and development of Electronics Systems as well as to interpret and synthesize experimental data leading to valid conclusions.
2. Exhibit the skills gathered to analyze, design, develop software applications and hardware products in the field of embedded systems and allied areas.

Program Educational Objectives (PEOs)

PEO1: Graduates exhibit their innovative ideas and management skills to meet the day to day technical challenges.

PEO2: Graduates utilize their knowledge and skills for the development of optimal solutions to the problems in the field of Electronics and Communication Engineering..

PEO3: Graduates exhibit good interpersonal skills, leadership qualities and adapt themselves for life-long Learning

PRINCIPLES OF COMMUNICATION SYSTEMS		Semester	4
Course Code	**BEC402**	CIE Marks	50
Teaching Hours/Week (L:T:P: S)	3:0:2:0	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 8-10 Lab slots	Total Marks	100
Credits	04	Exam Hours	03
Examination nature (SEE)	Theory/practical/Viva-Voce /Term-work/Others		

Course objectives:

This course will enable students to

- Understand and analyse concepts of Analog Modulation schemes viz; AM, FM
- Design and analyse the electronic circuits for AM and FM modulation and demodulation.
- Understand the concepts of random variable and random process to model communication systems.
- Understand and analyse the concepts of digitization of signals.
- Evolve the concept of SNR in the presence of channel induced noise

Teaching-Learning Process (General Instructions)

These are sample Strategies, which teacher can use to accelerate the attainment of the various course outcomes.

1. Lecture method (L) does not mean only traditional lecture method, but different type of teaching methods may be adopted to develop the outcomes.
2. Show Video/animation films to explain evolution of communication technologies.
3. Encourage collaborative (Group) Learning in the class.
4. Ask at least three HOTS (Higher order Thinking) questions in the class, which promotes critical thinking.
5. Adopt Problem Based Learning (PBL), which fosters students' Analytical skills, develop thinking skills such as the ability to evaluate, generalize, and analyze information rather than simply recall it.
6. Show the different ways to solve the same problem and encourage the students to come up with their own creative ways to solve them.
7. Discuss how every concept can be applied to the real world - and when that's possible, it helps improve the students' understanding.

MODULE-1

Random Variables and Processes: Introduction, Probability, Conditional Probability, Random variables. Statistical Averages: Function of a random variable, Moments, Random Processes, Mean, Correlation and Covariance function: Properties of autocorrelation function, Cross–correlation functions, Gaussian Process: Gaussian Distribution Function.

[Text 2: 5.1, 5.2,5.3,5.4,5.5,5.6,5.9]

RBT: L1, L2

MODULE-2

Amplitude Modulation Fundamentals: AM Concepts, Modulation index and Percentage of Modulation, Sidebands and the frequency domain, AM Power, Single Sideband Modulation.

AM Circuits: Amplitude Modulators: Diode Modulator, Transistor Modulator, collector Modulator. Amplitude Demodulators: Diode Detector, Balanced Modulators: Lattice Modulators.

Frequency Division Multiplexing: Transmitter-Multiplexer, Receiver-Demultiplexer.

[Text1: 3.1, 3.2,3.3,3.4,3.5,4.2,4.3,4.4,10.2]

RBT: L1, L2, L3

MODULE-3

Fundamentals of Frequency Modulation: Basic Principles of Frequency Modulation, Principles of Phase Modulation, Modulation index and sidebands, Noise Suppression Effects of FM, Frequency Modulation versus Amplitude Modulation.

FM Circuits: Frequency Modulators: Voltage Controlled Oscillators. , Frequency Demodulators: Slope Detectors, Phase Locked Loops.

Communication Receiver: Super heterodyne receiver, Frequency Conversion: Mixing Principles, JFET Mixer.

[Text1: 5.1,5.2,5.3,5.4,5.5,6.1,6.3,9.2,9.3]

RBT: L1, L2, L3

MODULE-4
Digital Representation of Analog Signals: Introduction, Why Digitize Analog Sources?, The Sampling process, Pulse Amplitude Modulation, Time-Division Multiplexing, Pulse Position Modulation: Generation and Detection of PPM wave. The Quantization Process. Pulse Code Modulation: Sampling, Quantization, Encoding, line Codes, Differential encoding, Regeneration, Decoding, filtering, multiplexing. **[Text2: 7.1,7.2,7.3,7.4,7.5,7.6,7.8,7.9]** **RBT: L1,L2,L3**
MODULE-5
Baseband Transmission of Digital signals: Introduction, Intersymbol Interference, Eye Pattern, Nyquist criterion for distortionless Transmission, Baseband M-ary PAM Transmission. **[Text2:8.1,8.4,8.5,8.6,8.7]** **Noise:** Signal to Noise Ratio, External Noise, Internal Noise, Semiconductor Noise, Expressing Noise Levels, Noise in Cascade Stages. **[Text1:9.5]** **RBT:L1,L2,L3**

PRACTICAL COMPONENT OF IPCC *(Experiments can be conducted using MATLAB/SCILAB/OCTAVE)*

Sl.NO	Experiments
1	Basic Signals and Signal Graphing: a) unit Step, b) Rectangular, c) standard triangle d) sinusoidal and e) Exponential signal.
2	Illustration of signal representation in time and frequency domains for a rectangular pulse.
3	Amplitude Modulation and demodulation: Generation and display the relevant signals and its spectrums.
4	Frequency Modulation and demodulation: Generation and display the relevant signals and its spectrums.
5	Sampling and reconstruction of low pass signals. Display the signals and its spectrum.
6	Time Division Multiplexing and demultiplexing.
7	PCM Illustration: Sampling, Quantization and Encoding
8	Generate a)NRZ, RZ and Raised cosine pulse, b) Generate and plot eye diagram
9	Generate the Probability density function of Gaussian distribution function.
10	Display the signal and its spectrum of an audio signal.

Course outcomes (Course Skill Set):

At the end of the course, the student will be able to:

1. Understand the principles of analog communication systems and noise modelling.
2. Identify the schemes for analog modulation and demodulation and compare their performance.
3. Design of PCM systems through the processes sampling, quantization and encoding.
4. Describe the ideal condition, practical considerations of the signal representation for baseband transmission of digital signals.
5. Identify and associate the random variables and random process in Communication system design.

Assessment Details (both CIE and SEE)

The weightage of Continuous Internal Evaluation (CIE) is 50% and for Semester End Exam (SEE) is 50%. The minimum passing mark for the CIE is 40% of the maximum marks (20 marks out of 50) and for the SEE minimum passing mark is 35% of the maximum marks (18 out of 50 marks). The student is declared as a pass in the course if he/she secures a minimum of 40% (40 marks out of 100) in the sum total of the CIE (Continuous Internal Evaluation) and SEE (Semester End Examination) taken together.

The IPCC means the practical portion integrated with the theory of the course. CIE marks for the theory component are **25 marks** and that for the practical component is **25 marks**.

CIE for the theory component of the IPCC

- 25 marks for the theory component are split into **15 marks** for two Internal Assessment Tests (Two Tests, each of 15 Marks with 01-hour duration, are to be conducted) and **10 marks** for other assessment methods mentioned in 22OB4.2. The first test at the end of 40-50% coverage of the syllabus and the second test after covering 85-90% of the syllabus.
- Scaled-down marks of the sum of two tests and other assessment methods will be CIE marks for the theory component of IPCC (that is for **25 marks)**.
- The student has to secure 40% of 25 marks to qualify in the CIE of the theory component of IPCC.

CIE for the practical component of the IPCC

- **15 marks** for the conduction of the experiment and preparation of laboratory record, and **10 marks** for the test to be conducted after the completion of all the laboratory sessions.
- On completion of every experiment/program in the laboratory, the students shall be evaluated including viva-voce and marks shall be awarded on the same day.
- The CIE marks awarded in the case of the Practical component shall be based on the continuous evaluation of the laboratory report. Each experiment report can be evaluated for 10 marks. Marks of all experiments' write-ups are added and scaled down to **15 marks**.
- The laboratory test **(duration 02/03 hours)** after completion of all the experiments shall be conducted for 50 marks and scaled down to **10 marks.**
- Scaled-down marks of write-up evaluations and tests added will be CIE marks for the laboratory component of IPCC for **25 marks**.
- The student has to secure 40% of 25 marks to qualify in the CIE of the practical component of the IPCC.

SEE for IPCC

Theory SEE will be conducted by University as per the scheduled timetable, with common question papers for the course (**duration 03 hours**)

1. The question paper will have ten questions. Each question is set for 20 marks.
2. There will be 2 questions from each module. Each of the two questions under a module (with a maximum of 3 sub-questions), **should have a mix of topics** under that module.
3. The students have to answer 5 full questions, selecting one full question from each module.
4. Marks scored by the student shall be proportionally scaled down to 50 Marks

The theory portion of the IPCC shall be for both CIE and SEE, whereas the practical portion will have a CIE component only. Questions mentioned in the SEE paper may include questions from the practical component.

- The minimum marks to be secured in CIE to appear for SEE shall be 10 (40% of maximum marks-25) in the theory component and 10 (40% of maximum marks -25) in the practical component. The laboratory component of the IPCC shall be for CIE only. However, in SEE, the questions from the laboratory component shall be included**.** The maximum of 04/05 sub-questions are to be set from the practical component of IPCC, the total marks of all questions should not be more than 20 marks.
- SEE will be conducted for 100 marks and students shall secure 35% of the maximum marks to qualify for the SEE. Marks secured will be scaled down to 50.
- The student is declared as a pass in the course if he/she secures a minimum of 40% (40 marks out of 100) in the sum total of the CIE (Continuous Internal Evaluation) and SEE (Semester End Examination) taken together.

Suggested Learning Resources:

Books

1. Louis E Frenzel, Principles of Electronic Communication Systems, 3rd Edition, Mc Graw Hill Education (India) Private Limited, 2016. ISBN: 978-0-07-066755-6.
2. Simon Haykin & Michael Moher, Communication Systems, 5th Edition, John Wiley, India Pvt. Ltd, 2010, ISBN: 978-81-265-2151-7.

Reference Books

1. B P Lathi, Zhi Ding, "Modern Digital and Analog Communication Systems", Oxford University Press., 4th edition, 2010, ISBN: 97801980738002.
2. Herbert Taub, Donald L Schilling, Goutam Saha, "Principles of Communication systems", 4th Edition, Mc Graw Hill Education (India) Private Limited, 2016. ISBN: 978-1-25-902985-1

Web links and Video Lectures (e-Resources):

1. Principles of Communication Systems **https://nptel.ac.in/courses/108104091**
2. Communication Engineering **https://nptel.ac.in/courses/117102059**

Activity Based Learning (Suggested Activities in Class)/ Practical Based learning

1. Assignments and test – Knowledge level, Understand Level and Apply level
2. Experiential Learning by using free and open source software's SCILAB or OCTAVE
3. Open ended questions by faculty, Open ended questions from students

Module 1.

Random variables and processes.

- Introduction.
- Probability.
- Conditional Probability.
- Random variables.
- Statistical averages.
- Function of a random variable.
- Moments.
- Random process
- Mean.
- Correlation and covariance function.
- Properties of autocorrelation function.
- Properties of Cross-Correlation functions.
- Gaussian process.
- Gaussian Distribution Function.

Module 1. RANDOM VARIABLES & PROCESSES.

Introduction:

* Random signals are present in every practical communication system.
* A signal is random, if it is not possible to predict its precise level in advance.
* Every speech signal is a burst of energy in random times.
  Major source of receiver noise is <u>thermal noise</u> which is due to random motion of Electrons.
* For a random signal detection of precise value is difficult. So we describe a statistical properties like Average power in random signal or average spectral distribution of power. This is done with a probability theory.
* Processing of random signal is a Random process and it consists of sample function.

Probability theory:-

* It gives the chance of getting the event. An experiment is repeated and the outcome may differ due to random phenomenon or chance mechanism.

Example: Tossing a coin for 'n' trial, if we get head for "m" times, then $\frac{m}{n}$ is probability of Head.

Let us consider an experiment where

1) The set of all possible outcomes of experiment is the sample space 'S'
2) An event may be an single sample point or set of sample points.
3) A single sample point is an Elementary event.
4) The entire sample space 'S' is sure event and Null set ϕ is impossible event.

Sample space 'S' may be discrete or continuous.

Example of Discrete events, Tossing coin or Dice.

Continuous events are measurement of voltage @ noise source.

Mutually exclusive events:

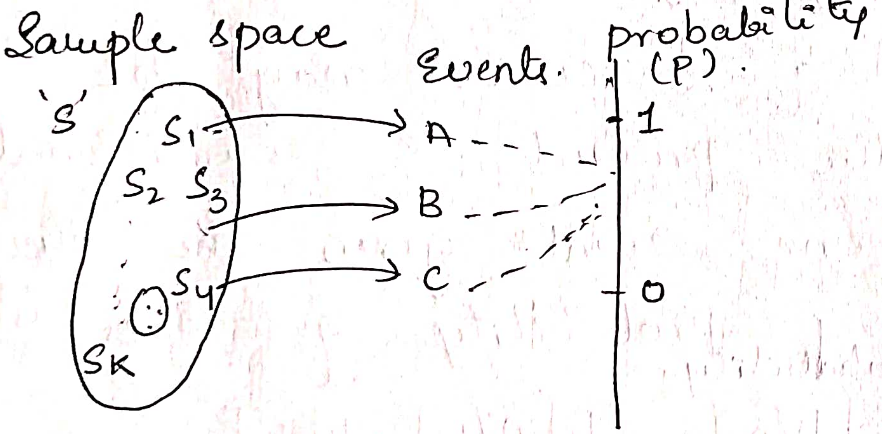
Two events are mutually exclusive, if occurance of one event preclude (stops) the occurance of other event.

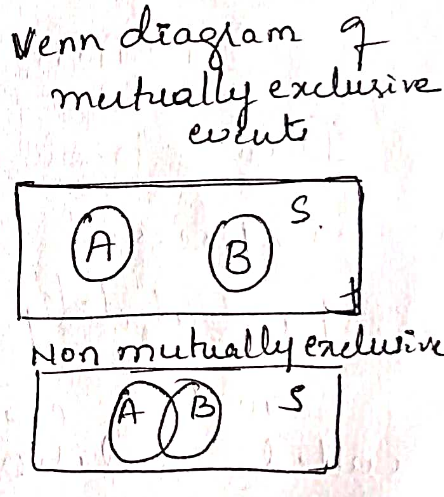
probability axioms (properties)

1) probability of any event will be b/w 0&1 ie $0 \leq P(A) \leq 1$.

2) $P[S] = 1$; ie Total probability of sample space is 1.

3) If 'A' & 'B' are 2 mutually exclusive events the $P[A \cup B] = P[A] + P[B]$





If P is area of space 'S' with S = 1.

then 1. $P[\bar{A}] = 1 - P[A]$.

2. $P[A \cup B] = P[A] + P[B] - P[A \cap B]$ ⟶ joint event

3) $P[A_1] + P[A_2] + P[A_3] + \ldots \ldots P[A_m] = 1$.

Conditional probability.

Suppose in an experiment a pair of outputs 'A' & 'B' are events.

Let $P[B/A]$ is probability of event 'B' given that 'A' has occured, then $P[B/A]$ is a conditional probability.

ie $P[B/A] = \frac{P[A \cap B]}{P[A]}$; where $P[A \cap B]$ is Joint probability

$\therefore P[A \cap B] = P[B/A]\, P[A]$

or

$P[A \cap B] = P[A/B]\, P[B]$.

Joint probability of 2 events may be expressed as the product of conditional probability of one event given the other and elementary probability of other is called conditional probability.

Baye's rule : $P[B/A] = \frac{P[A/B] \cdot P[B]}{P[A]}$

if $P[A] \neq 0$.

(Random variable is a rule, which assigns a real number to each possible outcomes)

Random variable:

A function whose Domain is a sample space and range is a set of real numbers is called as Random variable of the experiment.

Example: In Tossing a coin, Head corresponds to '1' and Tail to '0'.

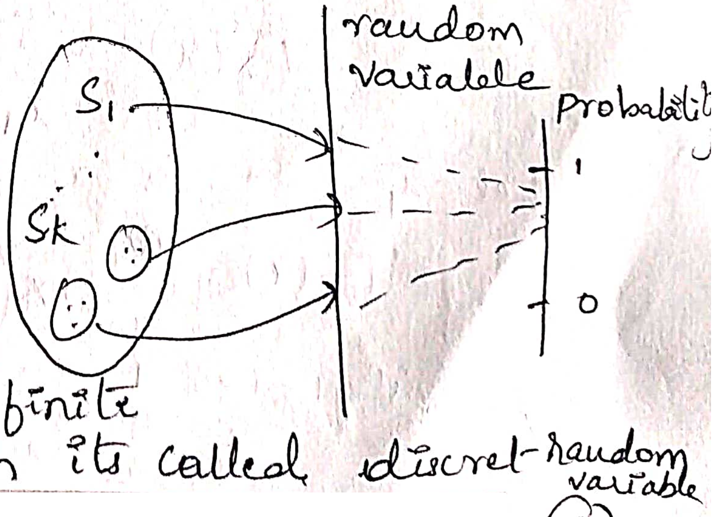
This numbers are assigned to outcomes of experiments.

∴ If the outcome of experiment is 's' then random variable is $X(s)$ or X..

The particular outcome of random experiment is $X(s_k) = x$.

Note: there may be more than one random variable associated with the same random experiment

Random variable can take Discrete values or Continuous values.



If Random variable takes only finite number of discrete values, then its called discret random variable

If a Random variables takes continuous values, then it is called continuous random variable.

Let 'X' be a random variable.

probability of event 'X' $\leq x$.

ie $P[X \leq x]$.

The function $\boxed{F_X(x) = P[X \leq x]}$

is a cumulative distribution function [CDF]

where 1) $F_X(x)$ is bounded b/w zero and one.

2) $F_X(x)$ is a monotone -non decreasing function x;

ie $F_X(x_1) \leq F_X(x_2)$ if $x_1 < x_2$.

The derivative of CDF, $F_X(x)$ is probability density function [PDF]

ie PDF, $\boxed{f_X(x) = \frac{dF_X(x)}{dx}}$.

$$P[x_1 \leq X \leq x_2] = P[X \leq x_2] - P[X \leq x_1]$$

$$= F_X(x_2) - F_X(x_1).$$

$$F_X(x) = \int_{x_1}^{x_2} f_X(x)\, dx.$$

Let $x_1 = -\infty$ & $x_2 = \infty$

$$F_X(x) = \int_{-\infty}^{\infty} f_X(\xi)\, d\xi$$

But $F_X(\infty) = 1$ and $F_X(-\infty) = 0$.

$$\therefore \int_{-\infty}^{\infty} f_X(x)\, dx = [F_X(x)]_{-\infty}^{\infty}$$

$$= F[\infty] - F[-\infty]$$

$$= 1 - 0 = \underline{\underline{1}}$$

Several Random variables:

Few experiments requires several random variables for description.

So if 'X' & 'Y' are two random variable.

Joint probability distribution is $F_{X,Y}(x,y)$.

where x, y are ~~sp~~ specified values.

Then o/p of experiment result is sample point lying inside

$(-\infty < X \leq x, -\infty < Y \leq y)$ of joint space.

ie $F_{X,Y}(x,y) = P[X \leq x, Y \leq y]$

Suppose Joint PDF is continuous then partial derivation

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{XY}(x,y)}{\partial x \, \partial y}$$ is a continuous.

Joint PDF is monotonic, Non decreasing function of both x and y.

The total value under graph of the joint probability density function must be unity as shown by

$$\int_{-\infty}^{\infty} f_{X,Y}(\xi,\eta)\, d\xi\, d\eta = 1.$$

The conditional probability density function of 'Y' given that $X = x$ is defined by

$$f_Y(y/x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

Statistical Average:

It is the expected value or mean of random variable 'X'

ie $\mu_X = E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx.$

Mean gives center of gravity of an area under probability density curve.

Joint probability density function:

- It is the PDF of two (or) more random variables.
- Joint PDF of two random variables X & Y is given by the partial derivative of the joint distribution function.

$$F_{XY}(x,y) = \frac{\partial^2 F_{XY}(x,y)}{\partial x \, \partial y}.$$

Function of a Random variable:

Let 'X' is a random variable with real-valued function $g(x)$, then expected value of Y with PDF $f_Y(y)$

$$Y = g(x).$$

$$E[Y] = \int_{-\infty}^{\infty} Y \cdot f_Y(y)\, dy.$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx.$$

Q) The random variable y is the function of another random variable 'x' such that $y = \cos(x)$. where x is a random variable uniformly distributed in interval $(-\pi, \pi)$.

ie
$$f_X(x) = \begin{cases} \frac{1}{2\pi} & -\pi < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

Find out the mean of Y

Solution:
$$E[Y] = \int_{-\pi}^{\pi} \cos x \frac{1}{2\pi}\, dx.$$

$$= \frac{1}{2}\left[\sin x\right]_{-\pi}^{\pi}$$

$$E[Y] = 0.$$

Moments:

- Let 'X' be a random variable with a real valued function $g(X) = x^n$ then n^{th} moment of probability distribution of X is

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x)\, dx.$$

First two moments are important.

for $n=1$, $E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$ is a mean of random variable

for $n=2$, $E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$ is a mean square value of 'X'

Central Moments:

It is moments of the difference b/w a random variable (X) and its mean $[\mu_X]$

$\therefore$ n^{th} central moment is

$$E[(x-\mu_X)^n] = \int_{-\infty}^{\infty} (x-\mu_X)^n f_X(x)\, dx.$$

for $n=1$, central moment is zero.

for $n=2$, 2^{nd} central moment is variance of random variable 'X'

ie $Var[X] = E[(x-\mu_X)^2] = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x)\, dx.$

- Variance is denoted as σ_x^2

Square root of the variance is standard deviation of random variable X.

$$SD = \sqrt{\text{variance}}$$
$$= \sqrt{\sigma_x^2}$$
$$= \sigma_x$$

$$SD = \sqrt{E[x^2] - \mu_x^2}$$

Random process:-

* when the random variable is a function of one independent variable (Time) it is called as random process.
* It is denoted by $x(t)$
* Random process or stochastic process is a sample space & ensemble

Difference b/w Random variable and Random process.

Random variable	Random process
1) It is a set of numbers.	1) It is a waveform.
2) It need not be function of time	2) It is the function of time.
3) Random variable are not further classified.	3) Random processes can be - stationery - Ergodic.
4) only ensemble average can be calculated	4) Ensemble as well as time average can be calculated

- Variance is denoted as σ_X^2.
- Square root of the variance.

$\sqrt{\sigma_X^2} = \sigma_X$ is standard deviation of random variable 'X'.

$$SD = \sqrt{\text{variance}}$$
$$= \sqrt{\sigma_X^2}$$
$$= \sigma_X.$$

$$\boxed{SD = \sqrt{E[X^2] - \mu_X^2}}$$

Mean:-

For a Random Process X(t), the mean of the process X(t) as the expectation of the random variable obtained by observing the process at some time 't' is given by

$$\mu_X(t) = E[X(t)].$$
$$= \int_{-\infty}^{\infty} x\, f_{X(t)}(x)\, dx.$$

where $f_{X(t)}(x)$ is PDF of process at time 't'

Note: The mean of stationary process is constant i.e. $\mu_X(t) = \mu_X$ for all t.

Auto correlation:

The auto correlation function of the process $X(t)$ as the expectation of the product of two random variables $X(t_1)$ and $X(t_2)$, obtained by observing $X(t)$ at times t_1 & t_2 respectively.

ie $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1)X(t_2)}(x_1, x_2)\, dx_1\, dx_2$$

where, $f_{X(t_1),X(t_2)}(x_1, x_2)$ is a joint probability density of the random variable $X(t_1)$ & $X(t_2)$.

The auto correlation function of a stationary random process depends only on the time difference $t_2 - t_1$ as $R_X(t_1, t_2) = R_X(t_2, t_1)$ for all t_1 & t_2

Auto - Covariance:

Auto-covariance function of a stationary random process $X(t)$ is

$$C_X(t_1, t_2) = E[(X(t_1) - \mu_X)(X(t_2) - \mu_X)]$$

$$= R_X(t_2 - t_1) - \mu_X^2$$

Properties of Auto-Correlation function:

Defining the auto-correlation function of stationary process $x(t)$ as

$$R_x(\tau) = E[x(t+\tau)\,x(t)] \text{ for all } t \quad \text{——— (1)}$$

(1) The mean square value of the process may be obtained from $R_x(\tau)$ by putting $\tau = 0$

$$\therefore R_x(0) = E[x(t+0)\,x(t)]$$

$$= E[x^2(t)].$$

(2) The auto-correlation function $R_x(\tau)$ is an even function of τ.

ie $R_x(\tau) = R_x(-\tau)$.

we can rewrite (1) as.

$$R_x(\tau) = E[x(t)\,x(t-\tau)]$$

(3) The auto-correlation function $R_x(\tau)$ has it maximum magnitude at $\tau = 0$.

ie $|R_x(\tau)| \le R_x(0)$.

Proof: consider Non negative quantity.

$$E[(x(t+\tau) \pm x(t))^2] \geq 0.$$

Expanding

$$E[X^2(t+\tau)] \pm 2 \cdot E[X(t+\tau)\, X(t)] + E[X^2(t)] \geq 0.$$

From property ①.

$$R_X(0) \pm 2 \cdot R_X(\tau) + R_X(0) \geq 0.$$

$$2R_X(0) \pm 2R_X(\tau) \geq 0.$$

$$-R_X(0) \leq R_X(\tau) \leq R_X(0)$$

Cross-correlation function:

Let us consider $X(t)$ and $Y(t)$ are 2 random process with auto-correlation function

$$R_X(t,u) \text{ and } R_Y(t,u).$$

The cross correlation function of $X(t)$ and $Y(t)$ is defined by.

$$R_{XY}(t,u) = E[X(t) \cdot Y(u)]$$

and

$$R_{YX}(t,u) = E[Y(t) \cdot X(u)]$$

If $X(t)$ & $Y(t)$ are stationary then

$$R_{XY}(t,u) = R_{XY}(\tau)$$

where $\tau = t-u$.

Properties of Cross-correlation:

1) Cross correlation function is not generally an even function of τ.

2) It does not have maximum at the origin.

3) It obeys certain symmetry $R_{xy}(\tau) = R_{yx}(-\tau)$.

Q) A random variable has PDF given by $f_X(x) = 2e^{-2x}$ for $x \geq 0$. Find the probability that it will take a value b/w 1 & 3.

Solution: $p(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_X(x)\, dx$

Here $x_1 = 1$ & $x_2 = 3$.

$$P(1 \leq x \leq 3) = \int_1^3 2e^{-2x}\, dx$$

$$= 2 \left.\frac{e^{-2x}}{-2}\right|_1^3$$

$$= -1\left(e^{-6} - e^{-2}\right)$$

$$= e^{-2} - e^{-6}$$

$$P(1 \leq x \leq 3) = 0.1328$$

Q) The PDF of a random variable is given as $f_X(x) = \begin{cases} K & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

where K is constant.

i) Sketch the PDF & determine value of K.

ii) If $a = -1$ & $b = 2$, calculate $P(|X| \le c)$ for $c = 1/2$

Solution:-

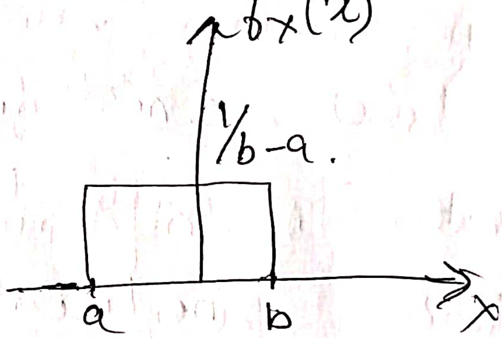
(1) w.k.t $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$.

$$= \int_a^b K\,dx = 1$$

$$= K\, x \Big|_a^b = 1$$

$$K(b-a) = 1$$

$$\boxed{K = \frac{1}{b-a}}$$



∴ PDF can be expressed as

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

ii) To determine $P(|X| \leq c)$ for $c = 1/2$.

ie $|x| < \frac{1}{2}$, $P(-1/2 \leq x \leq 1/2)$.

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} f_X(x)\, dx.$$

$$P(-1/2 \leq x \leq 1/2) = \int_{-1/2}^{1/2} \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\, x \Big|_{-1/2}^{1/2}.$$

$$= \frac{1}{b-a}\left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right]$$

$$= \frac{1}{b-a}\left[\frac{1}{2} + \frac{1}{2}\right]$$

$$P(-1/2 \leq x \leq 1/2) = \frac{1}{b-a}.$$

putting $a = -1$, $b = 2$.

$$P\left(-1/2 \leq x \leq \frac{1}{2}\right) = \frac{1}{2-(-1)}$$

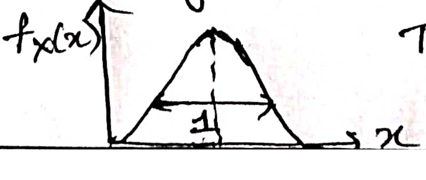
$$= \frac{1}{3}$$

Gaussian process:

* In Probability theory and statistics, a Gaussian process is a stochastic process (a collection of random variables indexed by time or space), such that every finite collection of those random variables has a multivariate normal distribution.

Gaussian distribution function:

* Gaussian distribution function is also known as normal distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence that data far from the mean.
* The graph of a Gaussian is a characteristic symmetric "bell curve" shape.



Total area = 1.

Properties of Gaussian PDF: (Probability distribution function)

Property 1:

* The gaussian (PDF) plot is symmetrical about mean value ($x=m$) (even symmetry)
$x=m$ is also the 'media' and 'mode' of the distribution.

$$\Rightarrow f_x(m-\sigma) = f_x(m+\sigma).$$

Property 2:

* The Peak of the Gaussian PDF is at its mean value ie at $x=m$.
So we can calulate this peak value $x=m$.

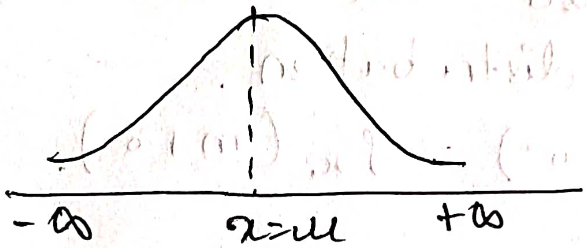
$$\Rightarrow f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \; ; \; x=m.$$

Property 3:

* The area under the Gaussian PDF curve below and above the mean value is 1/2

$P(X \leq m) = P(X \geq m) = \frac{1}{2}$.

The area under the curve is 1.



Total area = 1
$-\infty$ to x = 0.5
x to ∞ = 0.5

Property 4:

* For Gaussian distribution plot, its first derivative is positive for $x < m$ equal to 0 at $x = m$ & negative for $x > m$.