

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

JNANA SANGAMA, BELGAVI -590 014



“Control Systems (22BEC403)”

IPCC

(Theory / Practical)

(Effective from the academic Year 2024-2025)

Study Material for CS

EVEN SEMESTER – IV

Subject Code: 22BEC403

(Choice Based Credit System)

Prepared by:

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Department of Electronics& Communication Engineering **Akshaya Institute of Technology**

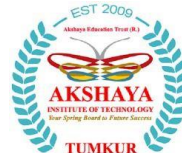
(Recognized by AICTE, New Delhi and Affiliated to Visvesvaraya Technological , University, Belagavi)

Akshaya Institute of Technology lingapura, Obalapura post, Koratagere Road,

Tumakuru-district-572106, Karnataka State, INDIA.



Akshaya Institute of Technology



(Recognized by AICTE, New Delhi and Affiliated to Visvesvaraya Technological , University, Belagavi)
Akshaya Institute of Technology lingapura, Obalapura post, Koratagere Road,
Tumakuru-district-572106, Karnataka State, INDIA.



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STUDENT'S NAME:

USN:

BRANCH:

SECTION: **YEAR:**

AKSHAYA INSTITUTE OF TECHNOLOGY

Lingapura, Obalapura Post, Koratagere Road, Tumakuru - 572106

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING



VISION

To produce competent engineering professionals in the field of Electronics and Communication Engineering by imparting value based quality technical education to meet the societal needs and to develop socially responsible citizens.



MISSION

M1: To provide strong fundamentals and technical skills in the field of Electronics and Communication Engineering through effective teaching learning process.

M2: Enhancing employability of the students by providing skills in the fields of VLSI, Embedded systems, Signal processing, etc., through Centre of Excellence.

M3: Encourage the students to participate in co-curricular and extra-curricular activities that creates a spirit of social responsibility and leadership qualities.



Program Specific Outcomes (PSOs)

After Successful Completion of Electronics and Communication Engineering Program Students will be able to

1. Apply fundamental knowledge of core. Electronics and Communication Engineering in the analysis, design and development of Electronics Systems as well as to interpret and synthesize experimental data leading to valid conclusions.
2. Exhibit the skills gathered to analyze, design, develop software applications and hardware products in the field of embedded systems and allied areas.



Program Educational Objectives (PEOs)

PEO1: Graduates exhibit their innovative ideas and management skills to meet the day to day technical challenges.

PEO2: Graduates utilize their knowledge and skills for the development of optimal solutions to the problems in the field of Electronics and Communication Engineering..

PEO3: Graduates exhibit good interpersonal skills, leadership qualities and adapt themselves for life-long Learning



Control Systems			
Course Code	BEC403	CIE Marks	50
Teaching Hours/Week (L: T: P)	(3:0:2)	SEE Marks	50
Total Hours of Pedagogy	40 hours Theory + 12 Lab slots	Total Marks	100
Credits	04	Exam Hours	03
Course objectives: This course will enable students to: 1. Understand basics of control systems and design mathematical models using block diagram reduction, SFG, etc. 2. Understand Time domain and Frequency domain analysis. 3. Analyze the stability of a system from the transfer function 4. Familiarize with the State Space Model of the system.			
Teaching-Learning Process (General Instructions) These are sample Strategies, which teacher can use to accelerate the attainment of the various course outcomes. • Lecture method (L) does not mean only traditional lecture method, but different type of teaching methods may be adopted to develop the outcomes. • Show Video/animation films to explain the different concepts of Linear Algebra & Signal Processing. • Encourage collaborative (Group) Learning in the class. • Ask at least three HOTS (Higher order Thinking) questions in the class, which promotes critical thinking. • Adopt Problem Based Learning (PBL), which fosters students' Analytical skills, develop thinking skills such as the ability to evaluate, generalize, and analyze information rather than simply recall it. • Topics will be introduced in a multiple representation. • Show the different ways to solve the same problem and encourage the students to come up with their own creative ways to solve them. • Discuss how every concept can be applied to the real world - and when that's possible, it helps improve the students' understanding. • Adopt Flipped class technique by sharing the materials / Sample Videos prior to the class and have discussions on the that topic in the succeeding classes. • Give Programming Assignments.			
Module-1			
Introduction to Control Systems: Types of Control Systems, Effect of Feedback Systems, Differential equation of Physical Systems -Mechanical Systems, Electrical Systems, Analogous Systems. (Textbook 1: Chapter 1.1, 2.2)			
Teaching-Learning Process	Chalk and Talk, YouTube videos RBT Level: L1, L2, L3		

Module-2	
Block diagrams and signal flow graphs: Transfer functions, Block diagram algebra and Signal Flow graphs. (Textbook 1: Chapter 2.4, 2.5, 2.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Any software tool to implement block diagram reduction techniques and Signal Flow graphs RBT Level: L1, L2, L3
Module-3	
Time Response of feedback control systems: Standard test signals, Unit step response of First and Second order Systems. Time response specifications, Time response specifications of second order systems, steady state errors and error constants. Introduction to PI, PD and PID Controllers (excluding design). (Textbook 1: Chapter 5.3, 5.4, 5.5)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Any software tool to show time response for various transfer functions and PI, PD and PID controllers. RBT Level: L1, L2, L3
Module-4	
Stability analysis: Concepts of stability, Necessary conditions for Stability, Routh stability criterion, Relative stability analysis: more on the Routh stability criterion. Introduction to Root-Locus Techniques, The root locus concepts, Construction of root loci. (Textbook 1: Chapter 6.1, 6.2, 6.4, 6.5, 7.1, 7.2, 7.3)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Any software tool to plot Root locus for various transfer functions RBT Level: L1, L2, L3
Module-5	
Frequency domain analysis and stability: Correlation between time and frequency response, Bode Plots, Experimental determination of transfer function. (Textbook 1: Chapter 4: 8.1, 8.2, 8.4) Mathematical preliminaries, Nyquist Stability criterion, (Stability criteria related to polar plots are excluded) (Textbook 1: 9.2, 9.3) State Variable Analysis: Introduction to state variable analysis: Concepts of state, state variable and state models. State model for Linear continuous –Time systems, solution of state equations. (Textbook 1: 12.2, 12.3, 12.6)	
Teaching-Learning Process	Chalk and Talk, YouTube videos, Any software tool to draw Bode plot for various transfer functions RBT Level: L1, L2, L3

Suggested Learning Resources:

Text Books

1. Control Systems Engineering, I J Nagrath, M. Gopal, New age international Publishers, Fifth edition.

List of Reference Books		
1. . "Modern Control Engineering" by Katsuhiko Ogata 2. "Automatic Control Systems" by Benjamin C. Kuo and Farid Golnaraghi		
List of URLs, Text Books, Notes, Multimedia Content, etc		
Control Systems https://nptel.ac.in/courses/108104091		
Course Outcomes: Students will be able to		
CO1: Model and analyze dynamic systems using mathematical tools.	L2	
CO2: Design control systems to achieve stability and performance criteria.	L2	
CO3: Implement various types of controllers such as PID and state feedback.	L3	
CO4: Use MATLAB/Simulink for simulation and analysis of control systems.	L3	

Web links and Video Lectures (e-Resources):
• https://nptel.ac.in/courses/108106098
Activity Based Learning (Suggested Activities in Class)/ Practical Based learning <i>Programming Assignments / Mini Projects can be given to improve programming skills</i>

CO5: Apply control system concepts to real-world engineering problems

L3

Subject Code:	BEC403	TITLE: Control Systems								Faculty Name:				
List of Course Outcomes	Program Outcomes												Total	
	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12		
CO-1	3	2	2	1	2	-	-	-	1	1	-	2	14	
CO-2	3	2	1	2	2	-	-	-	1	-	-	3	14	
CO-3	3	1	1	1	2	-	-	-	1	-	-	3	12	
CO-4	2	1	1	1	2	-	-	-	1	1	-	2	11	
CO-5	2	1	2	1	2	-	-	-	1	1	-	2	12	
Total	13	8	7	6	10	-	-	-	5	3	-	12	63	

Subject Code:	BEC403	TITLE: CS	
List of Course Outcomes	Program Specific Outcomes		Total
	PSO1	PSO2	
CO-1	2	1	3
CO-2	2	1	3
CO-3	3	2	5
CO-4	2	2	4
CO-5	2	1	3
Total	11	6	18

Note: 3 = Strong Contribution 2 = Average Contribution 1 = Weak Contribution - = No Contribute

MODULE -2

Transfer function:

The input- output relationship in a linear time invariant system is given by the transfer function. For a time invariant system, it is defined as the ratio of Laplace transform of the out to the Laplace transform of the input

The important features of the transfer functions are,

- The transfer function of a system is the mathematical model expressing the differential equation that relates the output to input of the system.
- The transfer function is the property of a system independent of magnitude and the nature of the input .
- The transfer function includes the transfer functions of the individual elements. But at the same time, it does not provide any information regarding physical structure of the system
- If the transfer function of the system is known, the output response can be studied for various types of inputs to understand the nature of the system
- It is applicable to Linear Time Invariant system.
- It is assumed that initial conditions are zero.
- It is independent of i/p excitation.
- It is used to obtain systems o/p response.
- If the transfer function is unknown, it may be found out experimentally by applying known inputs to the device and studying the output of the system

From the above block diagram $G(S) = C(S) / E(S)$ &

$$E(S) = R(S) - B(S)$$

$$\text{So, } C(S) = G(S) .E(S)$$

$$= G(S)[R(S)- B(S)]$$

$$= G(S) [R(S) - H(S).C(S)]$$

Therefore,

$$C(S)/R(S) = G(S) / [1+ G(S).H(S)]$$

This is the transfer function of the closed loop control system

Block Diagrammatic Representation

It is a representation of the control system giving the inter-relation between the transfer function of various components. The block diagram is obtained after obtaining the differential equation & Transfer

function of all components of a control system. The arrow head pointing towards the block indicates the i/p & pointing away from the block indicates the o/p.

Suppose $G(S)$ is the Transfer function then $G(S) = C(S) / R(S)$

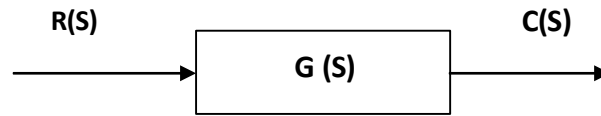


Fig.2.1. Block Diagram Representation of a Transfer Function

After obtaining the block diagram for each & every component, all blocks are combined to get a complete representation. It is then reduced to a simple form with the help of block diagram algebra.

Basic elements of a block diagram

- Blocks
- Transfer functions of elements inside the blocks
- Summing points
- Take off points
- Arrow

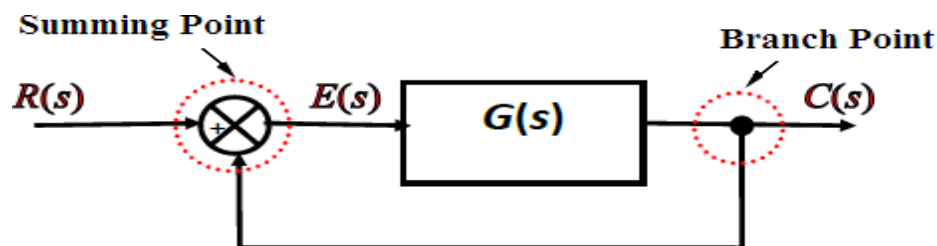


Fig 2.2. Basic elements of a Block Diagram

A control system may consist of a number of components. A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals. The elements of a block diagram are block, branch point and summing point.

(a) Block :

In a block diagram all system variables are linked to each other through functional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output.

(b) Summing point:

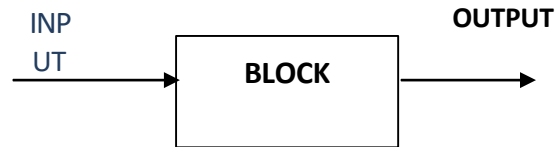


Fig.2.3 Block diagram

The blocks are used to identify many types of mathematical operations, like addition and subtraction and represented by a circle, called a summing point. As shown below diagram a summing point may have one or several inputs. Each input has its own appropriate plus or minus sign. A summing point has only one output and is equal to the algebraic sum of the inputs

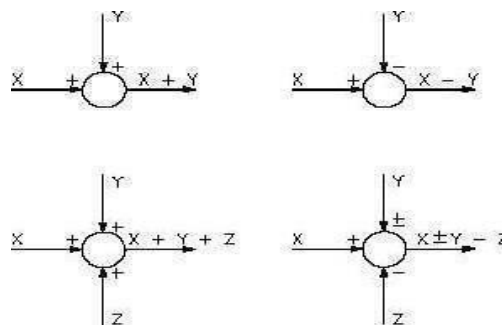


Fig.2.4 Summing Point

Individual & Overall performance can be studied A takeoff point is used to allow a signal to be used by more than one block or summing point

(c) Arrow – associated with each branch to indicate the direction of flow of signal

Advantages of Block Diagram Representation:

- It is always easy to construct the block diagram even for a complicated system
- Function of individual element can be visualized
- Over all transfer function can be calculated easily

Limitations of a Block Diagram Representation :

- No information can be obtained about the physical construction
- Source of energy is not shown

Block diagram reduction technique

Because of the simplicity and versatility, the block diagrams are often used by control engineers to describe all types of systems. A block diagram can be used simply to represent the composition and interconnection of a system. Also, it can be used, together with transfer functions, to represent the cause-and-effect relationships throughout the system. Transfer Function is defined as the relationship between an input signal and an output signal to a device.

Procedure to solve Block Diagram Reductions:

Step 1: Reduce the blocks connected in series

Step 2: Reduce the blocks connected in parallel Step

3: Reduce the minor feedback loops

Step 4: Try to shift take off points towards right and Summing point towards left

Step 5: Repeat steps 1 to 4 till simple form is obtained

Step 6: Obtain the Transfer Function of Overall System

1.5.1 Block diagram rules

(1) Blocks in Cascade [Series]: When two blocks are connected in series , their resultant transfer function is the product of two individual transfer functions.

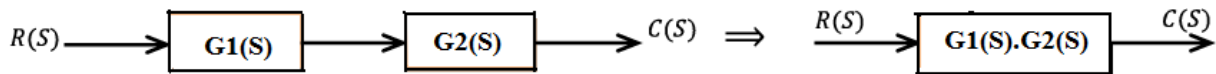


Fig. 2.5 Blocks in Cascade

(2) Combining blocks in Parallel: When two blocks are connected parallel as shown below, the resultant transfer function is equal to the algebraic sum (or difference) of the two transfer functions. This is shown in the diagram below.

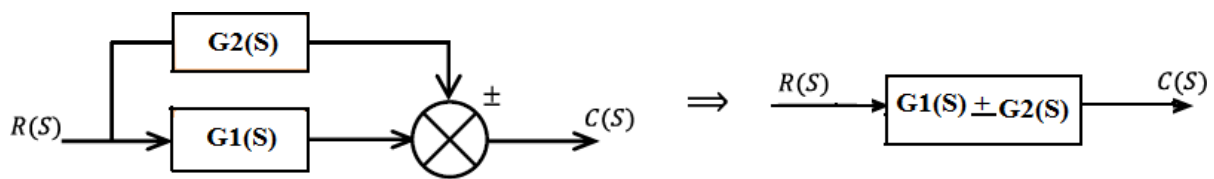


Fig. 2.6 combining blocks in Parallel

(3) Eliminating a feed back loop: The following diagram shows how to eliminate the feed back loop in the resultant control system

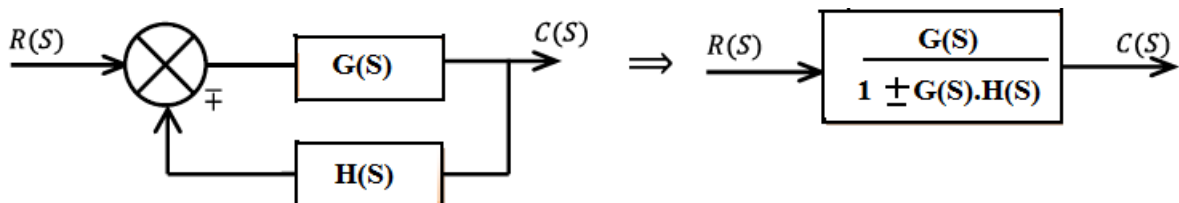


Fig. 2.7 Eliminating a feed back loop

(4) Moving a take-off point beyond a block: The effect of moving the takeoff point beyond a block is shown below.

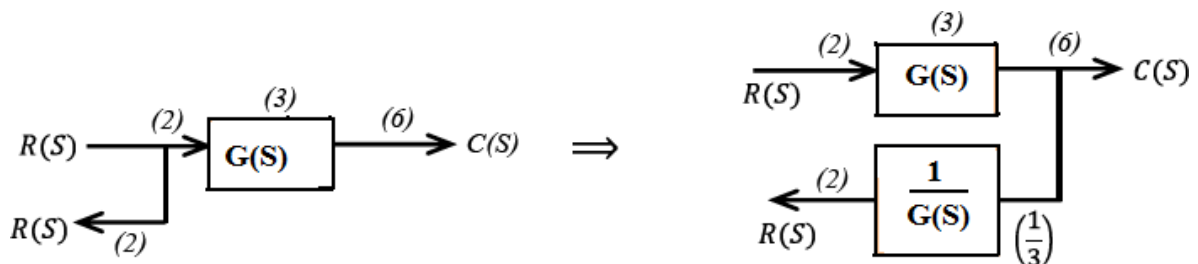


Fig. 2.8 Moving a take-off point beyond a block

(5) Moving a Take-off point ahead of a block: The effect of moving the takeoff point ahead of a block is shown below.

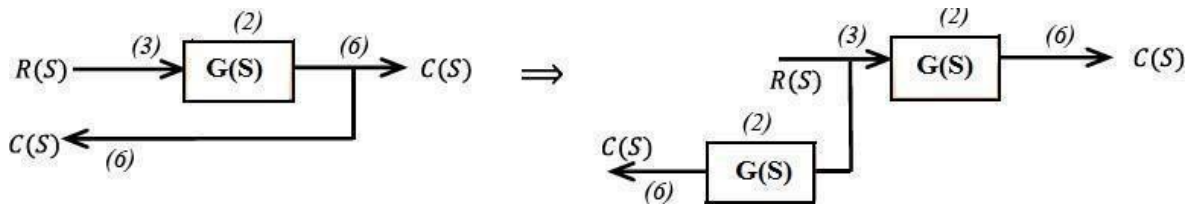


Fig: 2.9 Moving a take-off point beyond a block

Properties of Systems

For any control system to understand its performance the following properties are very important.

(i).Linearity: A system is said to be linear if it follows both the law of additivity and law of homogeneity. The system which does not follow the law of homogeneity and additivity is called a non-linear system.

If input $x_1(t)$ produces response $y_1(t)$ and input $x_2(t)$ produces response $y_2(t)$ then the scaled and summed input $a_1x_1(t) + b_1x_2(t)$ produces the scaled and summed response $a_1y_1(t) + b_1y_2(t)$ where a_1 and a_2 are real scalars. It follows that this can be extended to an arbitrary number of terms, and so for real numbers.

(ii) Time Invariance : A system with input $x(t)$ and output $y(t)$ is time-invariant if $x(t - t_0)$ creates output $y(t - t_0)$ for all inputs x and shifts t_0 .

(iii) Causality : A system is causal, if the output $y(t)$ at time t is not a function of future inputs and it depends only on the present and past inputs . All analog systems are causal and all memoryless systems are causal.

If the system is causal, then this implies $h(t) = 0, t < 0$. Alternatively, $h[n] = 0, n < 0$.

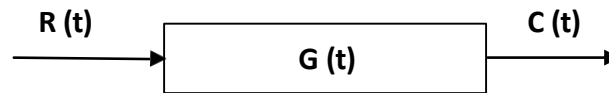
(iv) Stability : A system is said to be a stable if for every bounded-input there exists a bounded output

1.10 Transfer Function

For an open loop control system shown below the transfer function is the ratio of Laplace transform of the output to the Laplace transform of the input.

The Laplace transform of the Input is $R(S)$ and the Laplace transform of the output is

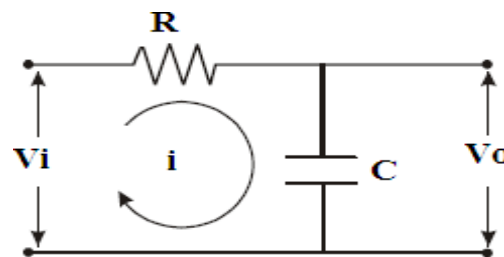
$C(S)$.



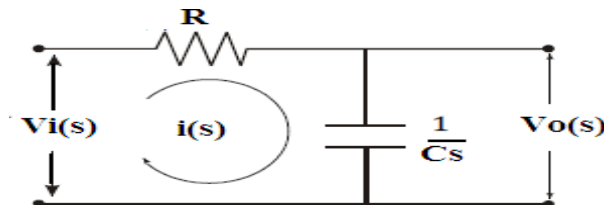
2.10 Transfer Function

So, the Transfer function of the system is $G(S) = C(S) / R(S)$

Example : Find the transfer function of the following RC circuit



2.10a: Transfer Function



2.10b: Transfer Function

The Laplace transformed Network is shown above. From the circuit we can write that

$$V_o(S) = \frac{1}{Cs} * I(S) \text{ \& } V_i(S) [R + \frac{1}{Cs}]$$

$$Cs * V_o(S) = V_i(S) / [R + \frac{1}{Cs}]$$

$$V_o(S) / V_i(S) = \frac{1}{\{ [R + \frac{1}{Cs}] \}}$$

$$Cs$$

$$V_o(S) / V_i(S) = \frac{1}{\{ [1 + RCs] \}} = \frac{1}{\{ 1 + \tau s \}} \text{ Where } \tau = RC$$

$$\text{Or the Transfer function } G(S) = \frac{1}{[1 + \tau s]}$$

Signal Flow Graphs (SFG)

The block diagram method is a useful tool for simplifying the representation of a control system. But when there are more than two feed back loops and if there exists inter-coupling between feedback loops, and when a system has more than one input and one output, the block diagram approach is very complex. Hence an alternate method is proposed by S.J. Mason. This method is called signal flow graphs. In these graphs each node represents a system variable & each branch connected between two nodes acts as Signal Multiplier. The direction of signal flow is indicated by an arrow.

A signal flow graph is a diagram that represents a set of simultaneous equations. It consists of a graph in which nodes are connected by directed branches. The nodes represent each of the system variables. A branch connected between two nodes acts as a one-way signal multiplier: the direction of signal flow is indicated by an arrow placed on the branch, and the multiplication factor (transmittance or transfer function) is indicated by a letter placed near the arrow.



So, in the figure above, the branch transmits the signal x_1 from left to right and multiplies it by the quantity a in the process. The quantity a is the transmittance, or transfer function.

Flow-Graph Definitions : A node performs two functions:

- Addition of the signals on all incoming branches and Transmission of the total node signal (the sum of all incoming signals) to all outgoing branches

There are three types of nodes. They are Source nodes, Sink nodes and Mixed nodes

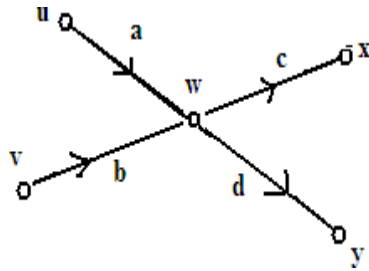
Source nodes (independent nodes) : These represent independent variables and have only outgoing branches, nodes u and v are source nodes.

Sink nodes (dependent nodes): These represent dependent variables and have only incoming branches.

In Fig (a), nodes x and y are sink nodes.

Mixed nodes (general nodes): These have both incoming and outgoing branches. In Fig. (a), node w is a mixed node. A mixed node may be treated as a sink node by adding an outgoing branch of unity transmittance, as shown in Fig (b), for the equation $x = au + bv$

and $w = cx = cau + cbv$



Fig(a)

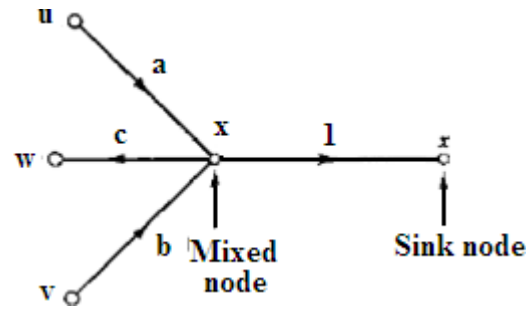


Fig (b)

A path is any connected sequence of branches whose arrows are in the same direction and A forward path between two nodes is one that follows the arrows of successive branches and in which a node appears only once. In Fig.(a) the path $uw x$ is a forward path between the nodes u and x .

Flow-Graph Algebra : The following rules are useful for simplifying a signal flow graph:

Series paths (cascade nodes). Series paths can be combined into a single path by multiplying the transmittances as shown in Fig (A).

Path gain. The product of the transmittances in a series path.

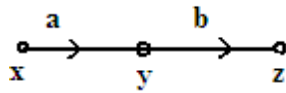
Parallel paths. Parallel paths can be combined by adding the transmittances as shown in Fig(B).

Feedback loop. A closed path that starts at a node and ends at the same node. Loop

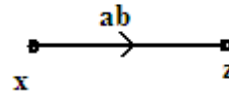
gain. The product of the transmittances of a feedback loop.

These results are shown diagrammatically in the following figures (A) ,(B) and C) where the original diagram and equivalent diagrams are shown.

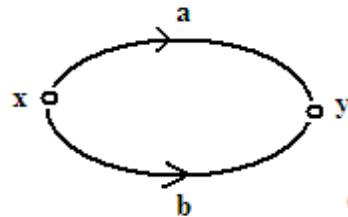
Original graph



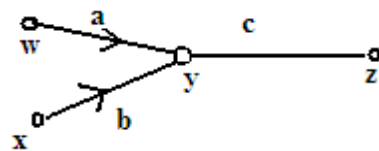
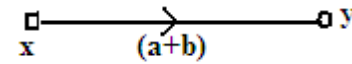
Equivalent graph



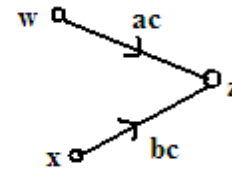
(A)



(B)



(C)



Masons gain formula

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between input and output nodes and is known as overall gain of the system. Mason's gain formula is used to obtain the overall gain (transfer function) of signal flow graphs. According to Mason's gain formula Gain is given by

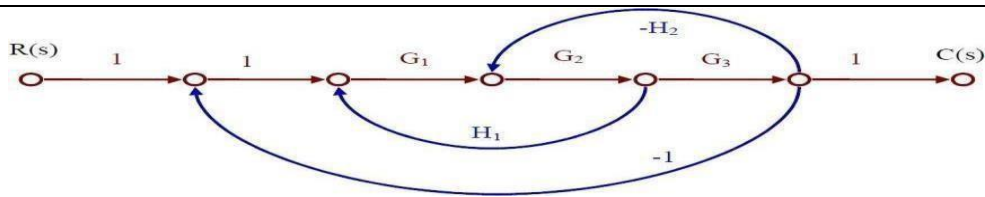
$$P = \frac{1}{\Delta} \sum_k \frac{P_k \Delta_k}{\Delta}$$

Where, P_k is gain of k^{th} forward path and Δ is determinant of graph. Here the Δ is given by

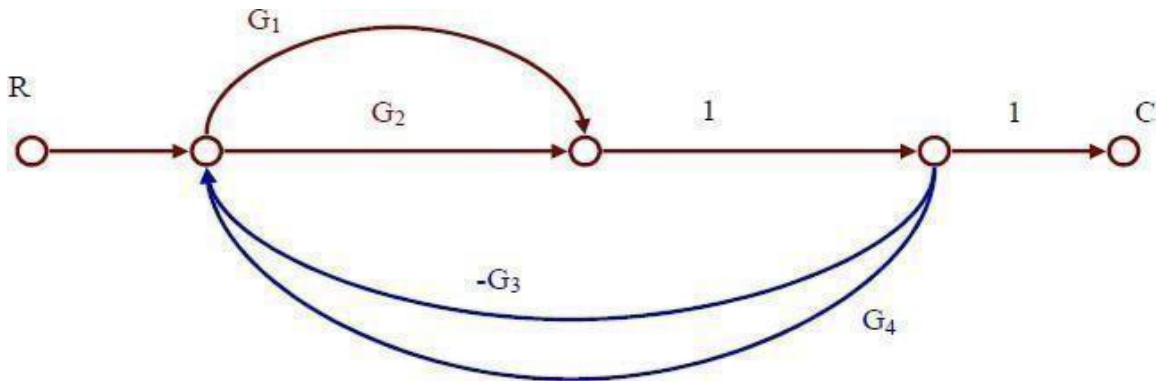
$\Delta = 1 - (\text{sum of all individual loop gains}) + (\text{sum of gain products of all possible combinations of two non touching loops}) - (\text{sum of gain products of all possible combination of three non touching loops})$
 Δ_k is cofactor of k^{th} forward path determinant of graph with loops touching k^{th} forward path. It is obtained from Δ by removing the loops touching the path P_k .

Finding transfer function from the system flow graphs is explained below by example.

Example1 : Obtain the transfer function of the system whose signal flow graph is shown



below.



There are two forward paths: One is Gain of path 1: $P_1=G_1$ and the other is Gain of path 2: $P_2=G_2$ There are four loops with loop gains:

$$L_1=-G_1G_3, L_2=G_1G_4, L_3= -G_2G_3, L_4= G_2G_4$$

There are no non-touching

loops. $\Delta = 1+G_1G_3-$

$$G_1G_4+G_2G_3-G_2G_4$$

Forward paths 1 and 2 touch all the loops. Therefore, $\Delta_1= 1, \Delta_2= 1$ So, the

transfer function T is given by

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 + G_1G_3 - G_1G_4 + G_2G_3 - G_2G_4}$$

Example 2 : Obtain the transfer function of $C(s)/R(s)$ of the system whose signal flow graph shown below.

From the system flow graph, it is clear that

There is one forward path, whose gain is: $P_1=G_1G_2G_3$ There

are three loops with loop gains:

$$L_1=-G_1G_2H_1, L_2=G_2G_3H_2, L_3= -G_1G_2G_3$$

There are no non-touching loops: $\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$

transfer function T is given by

$$\frac{C(S)}{R(S)} = \frac{P_1 \Delta_1}{1 - G_1 G_2 H_1 + G_1 G_3 H_2 + G_1 G_2 G_3}$$

System Stability:

The study of stability of a control system is very important to understand the performance. This means that the system must be stable at all times during operation. Stability may be used to define the usefulness of the system. Stability studies include absolute & relative stability. Absolute stability is the quality of stable or unstable performance. Relative Stability is the quantitative study of stability. The stability study is based on the properties of the Transfer Function. In the analysis, the characteristic equation is very important, which describe the transient response of the system. From the roots of the characteristic equation, following conclusions about the stability can be drawn.

(1) When all the roots of the characteristic equation lie in the left half of the S-plane, the system

response due to initial condition will decrease to zero at time. Thus the system will be termed as a stable system.

(2) When one or more roots lie on the imaginary axis & there are no roots on the RHS of

S-plane, the response will be oscillatory without damping. Such a system will be termed as critically stable.

(3) When one or more roots lie on the RHS of S-plane, the response will exponentially

increase in magnitude and thereby the system will be Unstable.