AKSHAYA INSTITUTE OF TECHNOLOGY
TUMKUR

Lecture Notes on

SUBECT : DISCRETE MATHEMATICAL STRUCTURES

SUBECT CODE : BCS405A

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AKSHAYA INSTITUTE OF TECHNOLOGY

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DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

VISION

To empower the students to be technically competent, innovative and self-motivated with human values and contribute significantly towards betterment of society and to respond swiftly to the challenges of the changing world.

MISSION

M1: To achieve academic excellence by imparting in-depth and competitive knowledge to the students through effective teaching pedagogies and hands on experience on cutting edge technologies.

M2: To collaborate with industry and academia for achieving quality technical education and knowledge transfer through active participation of all the stake holders.

M3: To prepare students to be life-long learners and to upgrade their skills through Centre of Excellence in the thrust areas of Computer Science and Engineering.

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Program Specific Outcomes (PSOs)

After Successful Completion of Computer Science and Engineering Program Students will be able to

* Apply fundamental knowledge for professional software development as well as to acquire new skills.
* Implement disciplinary knowledge in problem solving, analyzing and decision-making abilities through different domains like database management, networking, algorithms, and programming as well as research and development.
* Make use of modern computer tools for creating innovative career paths, to become an entrepreneur or desire for higher studies.

Program Educational Objectives (PEOs)

PEO1: Graduates expose strong skills and abilities to work in industries and research organizations.

PEO2 : Graduates engage in team work to function as responsible professional with good ethical behavior and leadership skills.

PEO3: Graduates engage in life-long learning and innovations in multi disciplinary areas.

MODULE 1: FUNDAMENTALS OF LOGIC

- Basics connectives and truth tables.
- Logic equivalence – The laws of logic.
- Logical implication – Rules of Inference.
- The use of Quantifiers.
- Definitions & proofs of theorems.

Proposition:

It is a statement or declaration which in a given content, can be either true or false but not both.

Example:

Banglore is in Karnataka (True)

- **Three is a Prime number (True)**
- **Seven is divisible by 3 (False)**
- **Every rectangle is a square (False)**

NOTE: **All sentence are not propositions.**

- Consider a triangle ABC
- xy = yx
- What an amazing day

Truth value :

The truth or falsity of a proposition is called its truth value.

- If proposition, p is true, its truth value is 1.
- if proposition, p is false, its truth value is 0.

Logical connectives:

The new propositions are obtained by using phrases like not, or, and,

if then, if and only if etc. such words or phrases are called logical connectives.

Compound propositions :

The new propositions obtained by the use of logical connectives are called compound propositions.

Simple propositions :

Propositions that do not contain any logical connective are called simple propositions.

Negation :

A proposition obtained by inserting word 'not' at an appropriate place in a given proposition is called the negation of the given proposition. It is denoted by ~p

Example:

p: 3 is an odd number.

~p: 3 is not an odd number

Conjunction :

A compound proposition obtained by combining two given propositions with ‘and’ in between them is called the conjunction of the given propositions. It is denoted by $p \wedge q$.

Example:

p: 3 is an odd number.

q: 3 +5 = 8

$p \wedge q$: 3 is an odd number and 3 +5 = 8

Disjunction :

A compound proposition obtained by combining two given propositions by inserting ‘or’ in between them is called the conjunction of the given propositions. It is denoted by $p \vee q$.

Example:

p: 3 is an odd number.

q: 9 is a prime number.

$p \vee q$: 3 is an odd number or 9 is a prime number.

Exclusive Disjunction :

If a compound proposition p or q is true, when either p or q is true but not both then it is said to be Exclusive Disjunction denoted by $p \underline{\vee} q$.

Example:

p: 3 is an odd number.

q: 2+3 =5

$p \underline{\vee} q$: Either 3 is an odd number or 2+3=5, but not both.

Conditional (Implication):

A compound proposition obtained by combining two propositions by the words 'if' and 'then' at appropriate places is called Conditional or Implication. It is denoted as $p \rightarrow q$ (if p then q)

Example:

p: I weigh more than 120 pounds.

q: I shall enroll in an exercise class

$p \rightarrow q$:If I weigh more than 120 pounds then I shall enroll in an exercise class.

Biconditional (Double Implication):

Let p and q are the two propositions, then the conjunctions (and) of the biconditional of p and q. it is denoted as $p \leftrightarrow q$

Example:

p: I weigh more than 120 pounds.

q: I shall enroll in an exercise class

$p \leftrightarrow q$:If I weigh more than 120 pounds then I shall enroll in an exercise class.

Tautology:

A Compound proposition which is always true regardless of the truth value of its components is called Tautology.

Contradiction:

A Compound proposition which is always false regardless of the truth value of its components is called contradiction or absurdity.

Contingency:

A Compound proposition that can be true or false is called contingency.

OR

contingency is a compound proposition which is neither a tautology nor a contradiction.

Logical equivalence:

Two propositions u and v are said to be logically equivalent whenever u and v have the same truth value or the biconditional $u \leftrightarrow v$ is always a tautology.

The logical equivalence is denoted by $\Leftrightarrow$.

Laws of Logic:

We recall that if $p \equiv q$, then the propositions p and q are said to be logically equivalent. This equivalence is called Law of logic.

Sl no	Law	Name of the law
1.	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Law
2.	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative law
3.	$p \wedge (q \wedge r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive law
4.	$\sim(p \wedge q) \equiv \sim p \wedge \sim q$ $\sim(p \vee q) \equiv \sim p \vee \sim q$	De-Morgan's law
5.	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption law
6.	$\sim(\sim p) \equiv p$	Double negation law

Sl no	Law	Name of the law
7.	$p \wedge p \equiv p$, $p \vee p \equiv p$	Idempotent Law
8.	$\sim(p \rightarrow q) \equiv p \wedge \sim q$	Negation of conditional
9.	$p \wedge T \equiv p$, $p \vee F \equiv p$	Identity law
10.	$p \wedge F \equiv F$, $p \vee T \equiv T$	Domination law
11.	$p \wedge \sim p \equiv F$, $p \vee \sim p \equiv T$	Inverse law

Duality of proposition:

Two propositions p and q involving basic connectives ∨, ∧, ~ are said to be duals of each other if replacement of ∨ by ∧ and ∧ by ∨(~ remains unchanged). Further if T by F and F by T.

NOTE:

1. $p \veebar q \Leftrightarrow (p \vee q) \wedge (\sim p \vee \sim q)$
2. $p \rightarrow q \Leftrightarrow \sim p \vee q$
3. $p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

Transitive and Substitution Rules :

1. If u, v, w are propositions such that $u \Leftrightarrow v$ and $v \Leftrightarrow w$, then $u \Leftrightarrow w$. This is known as the Transitive rule.
2. Suppose that a compound proposition u is a tautology and p is a component of u. If we replace each occurrence of p in u by a proposition q, then the resulting compound proposition v is also a tautology. This is called the Substitution rule.

Logical Implication:

Logical implication is a type of relationship between two statements or sentences. The relation translates verbally into "logically implies" or "if/then" and is symbolized by a double-lined arrow pointing toward the right ($\Rightarrow$). If p and q represent statements, then $p \Rightarrow q$ means "p implies q" or "If p, then q." The word "implies" is used in the strongest possible sense.

Example:

Suppose the sentences p and q are assigned as follows:

p = The sky is overcast.

q = The sun is not visible.

In this instance, $p \Rightarrow q$ is a true statement (assuming we are at the surface of the earth, below the cloud layer.) However, the statement

$p \Rightarrow q$ is not necessarily true; it might be a clear night. Logical implication does not work both ways. However, the sense of logical implication is reversed if both statements are negated. i.e., $(p \Rightarrow q) \Rightarrow (\sim q \Rightarrow \sim p)$

Necessary and Sufficient Conditions:

Consider two propositions p and q whose truth values are interrelated. Suppose that $p \Rightarrow q$. Then in order that q may be true it is sufficient that p is true. Also, if p is true then it is necessary that q is true. In view of this interpretation, all of the following statements are taken to carry the same meaning:

(i). $p \Rightarrow q$

(ii). p is sufficient for q

(iii). q is necessary for p

Rules of inference:

Let us consider the implication (p1 ∧ p2 ∧ …. ∧ pn) → q

Here n is a positive integer, the statements p1, p2, …. pn are called the **premises of the argument** and q is called the **conclusion of the argument**.

We write the above argument in the following tabular form:

$$\begin{array}{c} p_1 \\ p_2 \\ p_3 \\ \vdots \\ \underline{p_n} \\ \therefore q \end{array}$$

The preceding argument is said to be valid if whenever each of the premises p1, p2, …. pn is true, then the conclusion q is likewise true.

i.e., (p1 ∧ p2 ∧ …. ∧ pn) → q is valid when (p1 ∧ p2 ∧ …. ∧ pn) ⇒ q

Sl.no	Rules of inference	Name of rule
1	p $p \rightarrow q$ $\therefore q$	Rule of Detachment (modus pones)
2	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Law of Syllogism
3	$p \rightarrow q$ $\sim q$ $\therefore \sim p$	Modus Tollens
4	p q $\therefore p \wedge q$	Rule of Conjunction
5	$p \vee q$ $\sim p$ $\therefore q$	Rule of Disjunctive Syllogism
6	$\sim p \rightarrow Fo$ $\therefore p$	Rule of Contradiction
7	$p \wedge q$ $\therefore p$	Rule of Conjunctive Simplification
8	p $\therefore p \vee q$	Rule of Disjunctive Amplification

Open statement :

A declaration statement is an open statement

1. If it contains one or more variables.
2. If it is not statement.
3. But it becomes statement when the variables in it are replaced by certain allowable choices.

Example: "The number x+2 is an even integer" is denoted by P(x) then ¬ P(x) may be read as "The number x+2 is not an even integer".

Quantifiers:

The words “all”, “every”, “some”, “there exist” are associated with the idea of a quantity such words are called quantifiers.

Universal quantifiers:

The symbol ∀ has been used to denote the phrases “for all” and “for every” in logic “for each” and “for any” are also taken up to equivalent to these. These equivalent phrases are called universal quantifiers.

Existential quantifiers:

The symbol ∃ has been used to denote the phrases “there exist”, “for some” and “for at least one” each of these equivalent phrases is called the existential quantifiers.

Example:

1. For every integer x, x^2 is a non-negative integer $\exists\, x \in s, P(x)$.
2. For the universe of all integers, let

   p(x): $x>0$.

   q(x): x is even.

   r(x): x is a perfect square.

   s(x): x is divisible by 3

   t(x): x is divisible by 7.

Rules employed for determining truth value:

- **Rule1:** The statement “$\forall$ x $\in$ s, p(x)” is true only when p(x) is true for each x $\in$ s.
- **Rule2:** The statement “$\exists$ x $\in$ s, p(x)” is false only when p(x) is false for every x $\in$ s.
- **Rule3:** If an open statement p(x) is known to be true for all x in a universe s and if a $\in$ s then p(a) is true. (this is known as the rule of universal specification).
- **Rule4:** if an open statement p(x) is proved to be true for any (arbitrary) x chosen from a set s then the quantified statement $\forall$ x $\in$ s, p(x) is true.
- **Rule5:** To construct the negation of a quantified statement, change the quantifier from universal to existential and vice versa.

Methods of proof and methods of disproof:

Direct proof:

- **Hypothesis:** first assume that p is true.
- **Analysis:** starting with the hypothesis and employ the rules/ Laws of logic and other known facts infer that q is true.
- **Conclusion:** $p \rightarrow q$ is true.

Indirect proof:

- A conditional $p \rightarrow q$ and its contrapositive $\neg q \rightarrow \neg p$ is logically equivalent. In some situations, given a condition $p \rightarrow q$, a direct proof of the contrapositive $\neg q \rightarrow \neg p$ is easier. On the basis of this proof, we infer that the conditional $p \rightarrow q$ is true. This method of proving a conditional is called an indirect method of proof.

Proof by contradiction:

- **<u>Hypothesis</u>:** assume that p → q is false, that is assume that p is true and q is false.
- **<u>Analysis:</u>** starting with the hypothesis that q is false and employing the rules of logics and other known facts, this infer that p is false. This contradicts the assumption that p is true.
- **<u>Conclusion:</u>** because of the contradiction arrived in the analysis, we infer that p → q is true.

Proof by exhaustion:

Recall that a proposition of the form "∀ x ∈ S, p(x)" is true if p(x) is true for every x in S. if S consists of only a limited number of elements, we can prove that the statement "∀ x ∈ S, p(x)" is true by considering p(a) for each a in S and verifying that p(a) is true (in each case). Such a method of proof is called the method of exhaustion.

Disproof by counter example:

The way of disproving a proposition involving the universal quantifiers is to exhibit just one case where the proposition is false. This method of disproof is called disproof by counter example.