AKSHAYA INSTITUTE OF TECHNOLOGY

TUMKUR

Lecture Notes on

SUBECT : DISCRETE MATHEMATICAL STRUCTURES

SUBECT CODE : BCS405A

PREPARED BY : Dr. RASHMI S B M.SC., B.Ed., MISTE, Ph.D
ASSOCIATE PROFESSOR & HOD
DEPARTMENT OF MATHEMATICS
AKSHAYA INSTITUTE OF TECHNOLOGY
Tumkur

AKSHAYA INSTITUTE OF TECHNOLOGY

Lingapura, Obalapura Post, Koratagere Road, Tumakuru - 572106

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

VISION

To empower the students to be technically competent, innovative and self-motivated with human values and contribute significantly towards betterment of society and to respond swiftly to the challenges of the changing world.

MISSION

M1: To achieve academic excellence by imparting in-depth and competitive knowledge to the students through effective teaching pedagogies and hands on experience on cutting edge technologies.

M2: To collaborate with industry and academia for achieving quality technical education and knowledge transfer through active participation of all the stake holders.

M3: To prepare students to be life-long learners and to upgrade their skills through Centre of Excellence in the thrust areas of Computer Science and Engineering.

AKSHAYA
TUMKUR

Program Specific Outcomes (PSOs)

After Successful Completion of Computer Science and Engineering Program Students will be able to

* Apply fundamental knowledge for professional software development as well as to acquire new skills.
* Implement disciplinary knowledge in problem solving, analyzing and decision-making abilities through different domains like database management, networking, algorithms, and programming as well as research and development.
* Make use of modern computer tools for creating innovative career paths, to become an entrepreneur or desire for higher studies.

Program Educational Objectives (PEOs)

PEO1: Graduates expose strong skills and abilities to work in industries and research organizations.

PEO2 : Graduates engage in team work to function as responsible professional with good ethical behavior and leadership skills.

PEO3: Graduates engage in life-long learning and innovations in multi disciplinary areas.

MODULE 1: PROPERTIES OF INTEGERS

- **Mathematical induction**
- **The well ordering principle**
- **Recursive definitions**
- **Fundamental principles of counting**
- **Permutations and combinations**

Well ordering principle:

Every non-empty subset S of the set of positive integers N contains a least element. That is, there exists an element $m \in S$ such that $m \leq n$ for all $n \in S$.

That is, Every nonempty subset S of the positive integers contains a smallest element is said to be well ordered.

Principle of mathematical induction:

Consider a statement P(n), where n is a natural number. Then to determine the validity of P(n) for every n, use the following principle

- **Check whether the given statement is true for n = 1.**
- **Assume that given statement P(n) is also true for n = k, where k is any positive integer.**
- **Prove that the result is true for P(k+1) for any positive integer k.**

If the above-mentioned conditions are satisfied, then it can be concluded that P(n) is true for all n natural numbers.

EXAMPLES

1: Prove that the sum of cubes of n natural numbers is equal to $\left([n(n+1)]/2 \right)^2$ for all n natural numbers.

Solution:

In the given statement we have to prove that:

$$1^3+2^3+3^3+\cdots+n^3 = \left([n(n+1)]/2 \right)^2$$

Step 1:

Now with the help of the principle of induction in Maths, let us check the validity of the given statement P(n) for n=1.

$P(1)=([1(1+1)]/2)^2 = (2/2)^2 = 1^2 =1$.

This is true.

Step 2:

Now as the given statement is true for n=1, we shall move forward and try proving this for n=k, i.e.,

$$1^3+2^3+3^3+\cdots+k^3 = \left([k(k+1)]/2 \right)^2$$

Step 3:

Let us now try to establish that P(k+1) is also true.

$P(k+1) = ([k(k+1)]/2)^2$

So the result is true for n = k+1

By mathematical induction, the statement is true.

We see that the given statement is also true for n=k+1.

Hence we can say that by the principle of mathematical induction this statement is valid for all natural numbers n.

RECURSIVE DEFINITIONS

A sequence is an arrangement of any objects or a set of numbers in a particular order followed by some rule. If a1, a2, a3, a4,… etc. denote the terms of a sequence, then 1,2,3,4,… denotes the position of the term.

A sequence can be defined based on the number of terms i.e. either finite sequence or infinite sequence.

If $a_1, a_2, a_3, a_4, \ldots$ is a sequence, then the corresponding series is given by

$S_N = a_1+a_2+a_3 + \ldots + a_N$

Examples:

- **1, 2, 4, 7, 11, ...**
- **1, 1/2, 1/3, ...**
- **1, 8, 27, 64,…**

A sequence is represented by two methods

(i) Explicit methods

(ii) Recursive methods

Explicit methods:

Explicit formulas are algebraic expressions for a given term in a sequence, where the nth term is calculated through the index n.

If you need to make the formula based on a figure, you should try to separate the figure into smaller parts made up of known geometric shapes, like triangles, quadrilaterals, and so on.

Examples:

1. The sequence of odd numbers : 1,3,5,7,9,11,13…

The explicit formula for the nth term is then $a_n=2n-1$.

2. The sequence of triangular numbers is 1,3,6,10,15,21,...

The explicit formula appears by using the formula for the area of triangle $A=(b-h)/2$.

Recursive methods:

Recursive formulas express a term in a sequence through previous terms. The sequence decides what the recursive formula looks like

If you need to make the formula with a figure as the starting point, see how the figure changes and use that as a tool

Examples:

1. Fibonacci sequence is 1,1,2,3,5,8,13,21,34,55…

This sequence is itself recursive, because the previous terms decide what the next term is. You can see from this formula that any given term is dependent on the value of the previous two terms $a_{n+2} = a_{n+1}+a_n$ where $a_1=1, a_2=1$

2. The triangular numbers 1,3,6,10,15,21…

The above sequence develop by adding one new diagonal on the existing triangle. The recursive formula is $a_{n+1} = a_n+(n+1)$ where $a_1=1$. This formula comes from the fact that you add a row to the existing triangle, and this row always has n+1 dots.

$a_2=a_1+(1+1) = 1+2=3$

$a_3=a_2+(2+1) = 3+3=6$

Fundamental principle of counting:

The Fundamental Counting Principle, sometimes referred to as the fundamental counting rule, is a way to figure out the number of possible outcomes for a given situation.

- **Addition Principle (Rule of Sum)**
- **Multiplication Principle (Rule of Product)**

Addition Principle (Rule of Sum)

The Sum Rule states that if a task can be performed in either two ways, where the two methods cannot be performed simultaneously, then completing the job can be done by the sum of the ways to perform the task

Statement: **If there are n choices for one action, and m choices for another action and the two actions cannot be done at the same time, then there are n +m ways to choose one of these actions.**

Multiplication Principle (Rule of Product)

The Product Rule states that if a task can be performed in a sequence of tasks, one after the other, then completing the job can be done by the product of the ways to perform the task.

Statement: If there are n ways of doing something, and m ways of doing another thing after that, then there are n×m ways to perform both of these actions.

Example:

(1) Calvin wants to go to Milwaukee. He can choose from 33 bus services or 22 train services to head from home to downtown Chicago. From there, he can choose from 2 bus services or 3 train services to head to Milwaukee. How many ways are there for Calvin to get to Milwaukee?

He has 3+2=5 ways to get to downtown Chicago. (Rule of sum) From there, he has 2+3=5 ways to get to Milwaukee. (Rule of sum) Hence, he has 5×5=25 ways to get to Milwaukee in total. (Rule of product)

Permutation and Combination

Permutation: Permutation is a choice of things ‘r’, from a set of things ‘n’ without any replacement and also where an order matters

$$^{n}P_{r} = (n!) / (n-r)!$$

Combination: Combination is a choice of things ‘r’, from a set of things ‘n’ without any replacement but where order is not required and does not matter

$$^{n}C_{r} = {}^{n}P_{r} / r! = n! / \{r!\,(n-r)!\}$$

Example:

In how many ways a committee consisting of 5 men and 3 women, can be chosen from 9 men and 12 women?

Sol: Choose 5 men out of 9 men = $^9C_5$ ways = 126 ways

Choose 3 women out of 12 women = $^{12}C_3$ ways = 220 ways

Total number of ways = (126 x 220)= 27720 ways

The committee can be chosen in 27720 ways.

Binomial theorem expansion

Let $n \in N, x, y, \in R$ then $(x + y)^n = {}^n\Sigma_{r=0}\, nC_r\, x^{n-r} . y^r$ where,

${}^nC_r = {}^nP_r / r! = n! / \{r!\,(n-r)!\}$

Example:

1. $(\sqrt{2} + 1)^5 + (\sqrt{2} - 1)^5$

Sol: We have

$(x + y)^5 + (x - y)^5 = 2[5C_0\ x^5 + 5C_2\ x^3 y^2 + 5C_4\, xy^4]$

$= 2(x^5 + 10\, x^3 y^2 + 5xy^4)$

Now $(\sqrt{2} + 1)^5 + (\sqrt{2} - 1)^5 = 2[(\sqrt{2})^5 + 10(\sqrt{2})^3(1)^2 + 5(\sqrt{2})(1)^4]$

$=58\sqrt{2}$