AKSHAYA INSTITUTE OF TECHNOLOGY

Lingapura, Tumkur-Koratagere Road, Tumkur-572106.

To provide transformational technical competence by synergizing professional ethics and spiritual values to meet the global challenges and societal needs

Vision

DEPARTMENT OF PHYSICS

(FOR ALL ENGINEERING STREAM)

"APPLIED PHYSICS FOR CS STREAM" [BPHYS102/202]

Mission

- To impart value-based quality technical education nurture the students to adopt themselves to the ever-changing global needs
- To provide an experience the t inspires students to reach the highest level of accomplishment in their lives
- To provide an environment that enables students and faculty to make valuable contribution to the advancement of knowledge and creative practice of engineering

Prepared and verified by:

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Dr. Ashwini S Associate Professor
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DEPARTMENT OF PHYSICS

Course Title:	Applied Physics CS stream		
Course Code:	**BPHYS/102/202**	CIE Marks	50
Course Type (Theory/Practical/Integrated)	Integrated	SEE Marks	50
Teaching Hours/Week (L: T:P: S)	2:2:2:0	Exam Hours	03+02
Total Hours of Pedagogy	40 hours Theory + 10-12 Lab slots	Credits	04

COURSE OUTCOMES OF CS STREAM

CO1	**Describe** the principles of LASERS and Optical fibers and their relevant applications
CO2	**Discuss** the basic principles of Quantum Mechanics and their application in Quantum Computing
CO3	**Summarize** the essential properties of superconductors and applications in Quantum Computing.
CO4	**Illustrate** the application of physics in design and data analysis
CO5	**Practice** working in groups to conduct experiments in physics and **perform** precise and honest measurements

SYLLABUS OF 2022 SCHEME, CS STREAM

Module-1 (8 Hours): Laser and Optical Fibers:

LASER: Basic properties of a LASER beam, Interaction of Radiation with Matter, Einstein's A and B Coefficients, Laser Action, Population Inversion, Metastable State, Requisites of a laser system, Semiconductor Diode Laser, Applications: Bar code scanner, Laser Printer, Laser Cooling. Numerical problems.

Optical Fiber: Principle and structure, Acceptance angle and Numerical Aperture (NA) and derivation of Expression for NA, Classification of Optical Fibers, Attenuation and Fiber Losses, Applications: Fiber Optic networking, Fiber Optic Communication. Numerical Problems.

Module-2 (8 Hours): Quantum Mechanics:

de Broglie Hypothesis and Matter Waves, de Broglie wavelength and derivation of expression by analogy, Phase
Velocity and Group Velocity, Heisenberg's Uncertainty Principle and its application (Nonexistence of electron inside the nucleus-Non-Relativistic), Principle of Complementarity, Wave Function, Time independent Schrodinger wave equation, Physical Significance of a wave function and Born Interpretation, Expectation value, Eigen functions and Eigen Values, Particle inside one-dimensional infinite potential well, Waveforms and Probabilities. Numerical problems.

Module-3 (8 Hours): Quantum Computing:

Wave Function in Ket Notation: Matrix form of wave function, Identity Operator, Determination of I|0> and I|1>, Pauli Matrices and its operations on 0 and 1 states, Mention of Conjugate and Transpose, Unitary Matrix U, Examples: Row and Column Matrices and their multiplication (Inner Product), Probability, Orthogonality
Principles of Quantum Information & Quantum Computing: Introduction to Quantum Computing, Moore's law & its end. Single particle quantum interference, Classical & quantum information comparison. Differences between classical & quantum computing, quantum superposition and the concept of qubit.
Properties of a qubit: Mathematical representation. Summation of probabilities, Representation of qubit by Bloch

Module-4 (8 Hours): Electrical Properties of Materials and Applications

Electrical conductivity in metals, Resistivity and Mobility, Concept of Phonon, Matthiessen's rule. Introduction to Super Conductors, Temperature dependence of resistivity, Meissner's Effect, Silsbee Effect, Types of Superconductors, Temperature dependence of critical field, BCS theory (Qualitative), Quantum Tunneling, High- Temperature superconductivity, Josephson Junction, DC and AC SQUIDs (Qualitative), Applications in Quantum Computing (Mention). Numerical problems.

Module-5 (8 hours): Applications of Physics in computing:

Physics of Animation: Taxonomy of physics-based animation methods, Frames, Frames per Second, Size and Scale, weight and strength, Motion and Timing in Animations, Constant Force and Acceleration, The Odd rule, Motion Graphs, Numerical Calculations based on Odd Rule, Examples of Character Animation: Jumping, Walking. Numerical problems.
Statistical Physics for Computing: Descriptive statistics and inferential statistics, Poisson distribution and Normal Distributions (Bell Curves), Monte Carlo Method. Numerical problems.

K.S. Chandrashekar, M.Sc., M.Phil.,
Lecturer in Physics

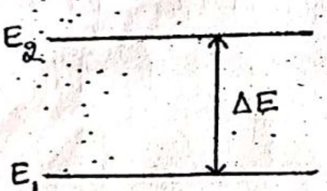
UNIT-III : LASERS.

The word LASER stands for "Light amplification by stimulated emission of radiation". Laser was invented by an American scientist 'Theodore Maiman' in 1960. A Laser produces a thin, high intense, a highly coherent parallel beam of light and its production is a particular consequence of interaction of radiation with matter.

Principle & production of Lasers.

The basic principle of Lasers is based on the phenomenon of "interaction of radiation with matter".

To understand the manner in which radiation can interact with matter, consider two energy levels E_1 & E_2 of a system with energy difference ΔE.



$\Delta E = E_2 - E_1$, $(E_2 > E_1)$

E_1 = Lower energy state of atom

E_2 = Higher energy state of atom

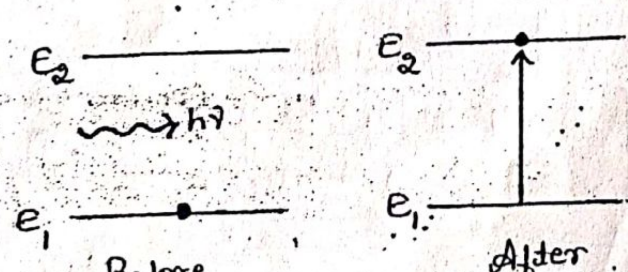
If a light of energy $E = h\nu$ falls on this system, there are three possible ways through which interaction of radiation with matter takes place. They are

① Induced Absorption:

The transition of atom from lower energy state (E_1) to higher energy state (E_2) by absorbing a photon of energy $(h\nu)$ is called "Induced Absorption".

The process is represented as $A + Ph \rightarrow A^*$

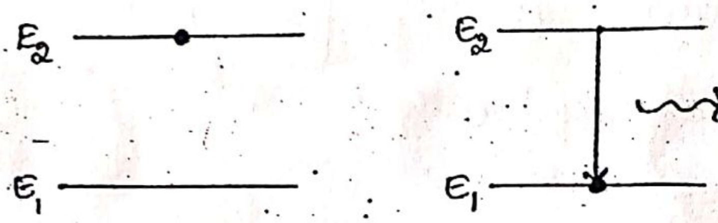
(Atom) + (photon) → (Atom)*



② Spontaneous Emission :

The transition of atom from higher energy state to (E_2) lower energy state (E_1) by emitting a photon of energy ($h\nu = E_2 - E_1$) without the influence of any photon of energy is called "Spontaneous Emission".

The process is represented as $A^* \rightarrow A + Ph$.



Before After

③ Stimulated Emission:

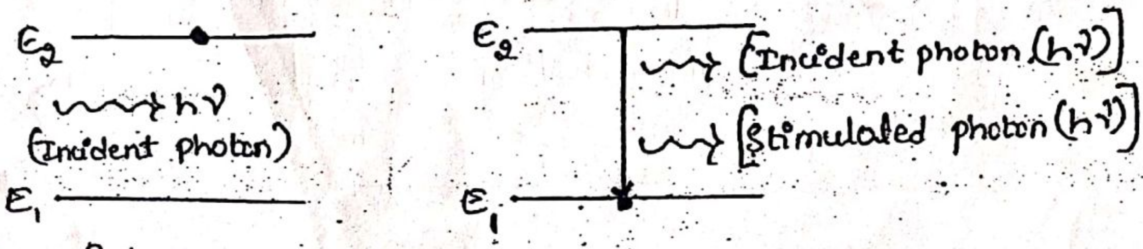
Consider the atom in the higher energy state (E_2). If the photon of energy ($h\nu$) incident on it, then the incident photon interacts with the atom, the atom jumps to a lower energy state by emitting additional photon along with the incident photon. This process is called "Stimulated emission".

[OR]

The transition of atom from higher energy state (E_2) to lower energy state (E_1) by a photon (stimulated photon) under the influence of a passing photon of energy ($h\nu = E_2 - E_1$) is called "Stimulated emission". The process is represented as

$$A^* + ph \longrightarrow A + (\text{photon} + \text{photon}).$$

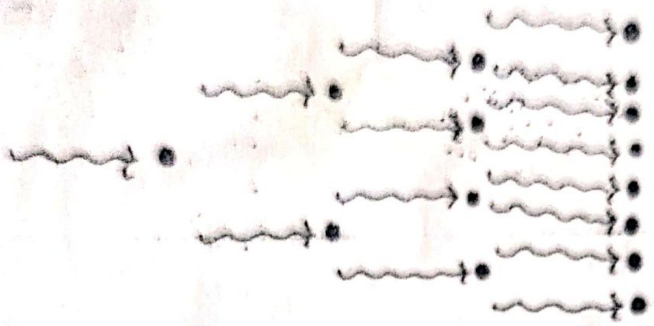
If the two photons emitted are identical in energy, phase, and travel in the same direction. This process is responsible for "Laser action".



Before After

If there two coherent photons then interact with two more excited state atoms, four coherent photons are produced and so on.

Therefore, the stimulated emission leads to photoamplification and is as shown below.



Boltzmann's ratio

Consider the number of atoms per unit volume that exist in a given energy state. This number is called population, 'N' given by Boltzmann's equation

$$\boxed{N = e^{-(E/KT)}}$$

where,

$E \rightarrow$ Energy level of the system

$K \rightarrow$ Boltzmann's constant

$T \rightarrow$ Absolute temperature.

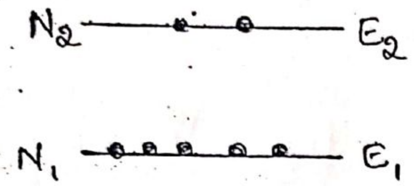
Consider a two energy state quantum system. Let N_1 & N_2 be the population of atoms in the lower state E_1 & higher state E_2 respectively. The ratio of populations in these two state, (N_2/N_1) is called Boltzmann's ratio (or relative population) given by

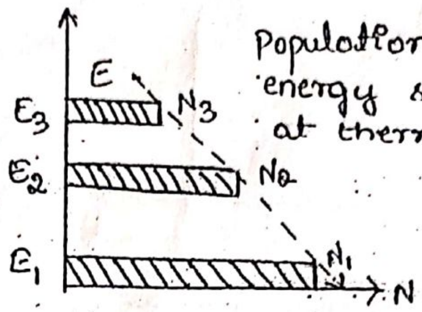
$$\frac{N_2}{N_1} = \frac{e^{-E_2/KT}}{e^{-E_1/KT}}$$

$$\boxed{\frac{N_2}{N_1} = e^{-(E_2-E_1)/KT}} \quad \text{------} (1)$$

Note: Eqⁿ ① $\Rightarrow$ since $E_2 > E_1$, RHS < 1, i.e $\boxed{N_2 < N_1}$

At thermal equilibrium, the system always tend to attain minimum energy state as possible i.e, more number of atoms in the lower state compared to higher state (figure).





Population of three energy state systems at thermal equilibrium

*** (6-8m)

① **Einstein Co-efficients** [ⓞⓡ Expression for energy density of radiation under thermal Equilibrium condition in terms of Einstein's co-efficients]

Consider two energy states E_1 & E_2 of a system of atoms ($E_2 > E_1$). Let N_1 be the population of atoms in the lower state and N_2 be the population of atoms in the higher state. Let $u(\nu)$ be the energy density of a system of frequency ν. ⓞⓡ $u(\nu)$ be the incident energy / unit volume.

Let us consider the following process one by one.

(a) **Induced absorption:**

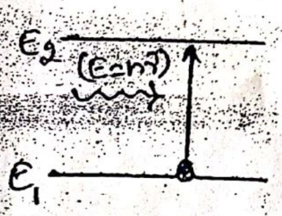
In this case, a certain number of atoms absorb energy ($h\nu$) and transit to the higher state.

The rate of absorption depends on

i) Number density of lower energy state N_1.

ii) the energy density $u(\nu)$

$\therefore$ Rate of absorption $\propto N_1 \cdot u(\nu)$



$$P_{12} = B_{12} N_1 \cdot u(\nu) \quad \text{------- (1)}$$

where, $B_{12} \rightarrow$ Einstein co-efficient of induced absorption

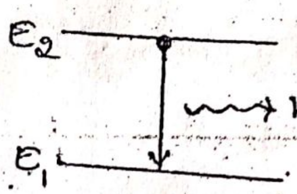
$P_{12} \rightarrow$ Rate of absorption

(ii) <u>Spontaneous emission:</u>

In this case, a certain number of atoms in the higher state lose energy of $h\nu$ without any external agency and transit to the lower state.

The rate of spontaneous emission depends only on number density in the higher energy state i.e., N_2.

Rate of spontaneous emission $\propto N_2$



$$\boxed{P_{21} = A_{21} \cdot N_2} \quad \text{------- (2)}$$

where, $A_{21} \rightarrow$ Einstein co-efficient of spontaneous emission.

$P_{21} \rightarrow$ Rate of spontaneous emission.

(iii) <u>Stimulated emission:</u>

In this case, a certain number of atoms in the higher state transition to the lower state by emission of stimulated photons.

The rate of stimulated emission depends on

i) Number density of the higher energy state i.e., N_2

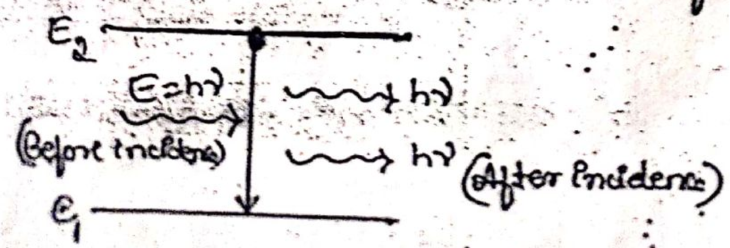
ii) Energy density i.e., $u(\nu)$

$\therefore$ Rate of stimulated emission $\propto N_2 \cdot u(\nu)$

$$\boxed{P'_{21} = B_{21} N_2 \cdot u(\nu)} \quad \text{------- (3)}$$

where, $B_{21} \rightarrow$ Einstein co-efficient of stimulated emission

$P'_{21} \rightarrow$ Rate of stimulated emission.



∴ At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

from Eqⁿ (1), (2) & (3)

$$B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu)$$

$$B_{12} N_1 u(\nu) - B_{21} N_2 u(\nu) = A_{21} N_2 \quad (\div \text{ by } N_2 \text{ on B.S})$$

$$\Rightarrow B_{12} \frac{N_1}{N_2} u(\nu) - B_{21} u(\nu) = A_{21}$$

$(\div$ by B_{21} on B.S$)$

$$\Rightarrow \frac{B_{12}}{B_{21}} \frac{N_1}{N_2} u(\nu) - u(\nu) = \frac{A_{21}}{B_{21}}$$

$$u(\nu)\left[\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1\right] = \frac{A_{21}}{B_{21}}$$

(or)

$$u(\nu) = \frac{A_{21}/B_{21}}{\left[\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1\right]} \quad \text{------ (4)}$$

By Boltzmann's law,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-h\nu/kT}$$

$$\therefore \frac{N_1}{N_2} = e^{h\nu/kT}$$

$$\therefore \text{Eq}^n \ (4) \Rightarrow u(\nu) = \frac{A_{21}/B_{21}}{\left[\frac{B_{12}}{B_{21}} e^{h\nu/kT} - 1\right]}$$

(or)

$$u(\nu) = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}} e^{h\nu/kT} - 1}\right] \quad \text{------ (5)}$$

According to planck's law.

Energy density, $u(\nu) = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{h\nu/kT} - 1}\right]$ ---- (6)

Comparing eq^n (5) & (6), we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} = \frac{A}{B}$$

& $\frac{B_{12}}{B_{21}} = 1$ (or) $\therefore B_{12} = B_{21}$

$\therefore$ It implies that the probability of induced absorption is equal to the probability of stimulated emission and A_{21} and B_{21} is simply represented as 'A' & 'B'.

$$\text{eq}^n \ (6) \Rightarrow \boxed{u(\nu) = \frac{A}{B\left[e^{h\nu/kT} - 1\right]}}$$

The above equation represents the energy density under thermal equilibrium in terms of Einstein's co-efficients

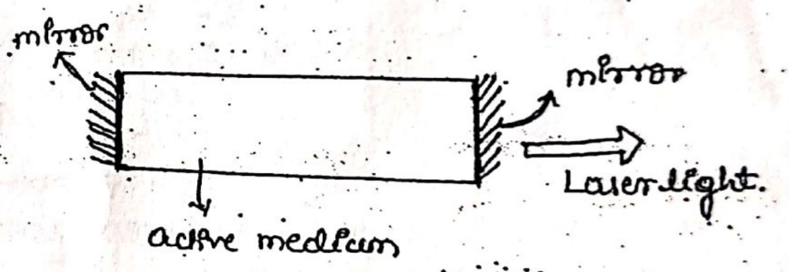
Production of Lasers.

Pumping: The process of exciting atoms from lower state to higher state by supplying energy from external source is called Pumping. There are different ways of pumping namely optical pumping, gas discharge, chemical reaction etc.

Lasing: The process of emission of stimulated photons from a quantum system after attaining population inversion is called as Lasing.

Active medium: The quantum system in which pumping & lasing takes place is called active medium.

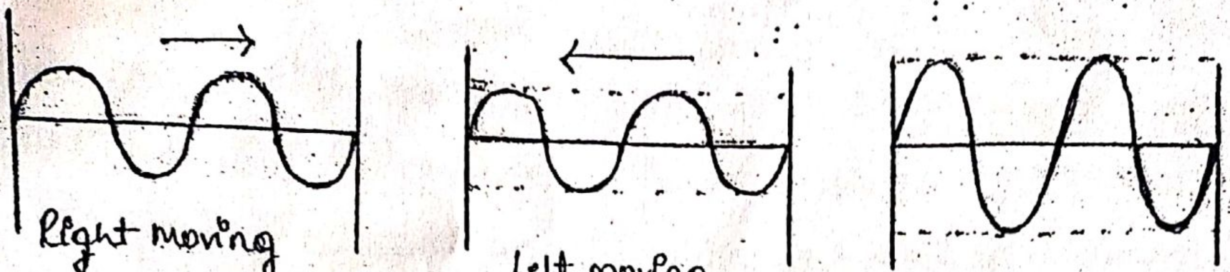
Laser cavity:



A Laser device consists of an active medium bound between two mirrors. The mirrors reflects the photons to and fro through active medium.

A photon moving in a particular direction represents a light wave moving in the same direction. Thus the two mirrors along with the active medium form a cavity and is called "Laser cavity".

Inside the laser cavity, there are two types of waves one type of wave moving to the right & other type moving to the left.



To get Constructive Interference, the distance between the mirror should be (L) an Integral multiple of $\lambda/2$.

i.e., $L = \frac{m\lambda}{2}$ $m \rightarrow$ an Integer (>0)

$\lambda \rightarrow$ wave length of laser inside the active medium.

In this case, a standing wave pattern is established within the cavity & the cavity is said to be resonant at wave lengths.

$$\boxed{\lambda = \frac{2L}{m}}$$

Requisites of a Laser system:

1) There must be at least a pair of energy levels (say $E_2 > E_1$) seperated by the radiation which is to be stimulated.

2) It requires an energy source.

3) It requires an active medium for population inversion.

4) It requires an excitation source for pumping action.

5) It requires an Laser Cavity which provides multiplica-tion of stimulated photons.

(2) Condition for Laser Action :

Population Inversion and metastable state are the conditions for laser action.

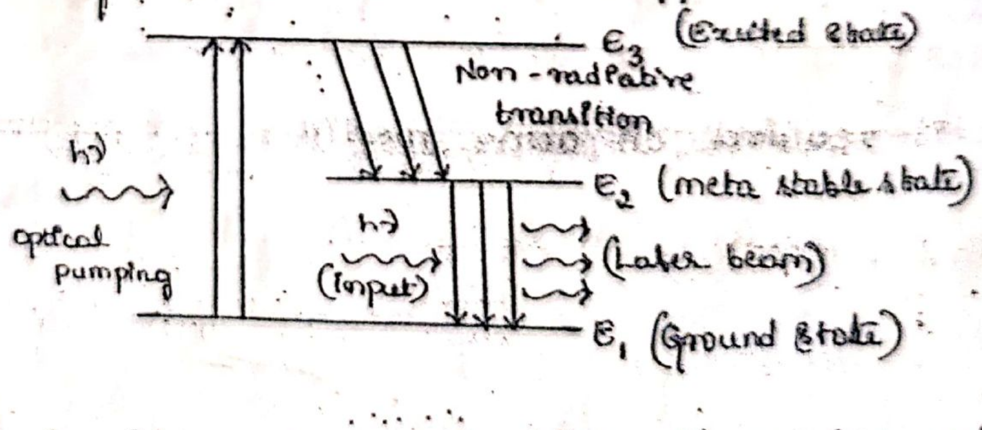
i) Population Inversion : It is the condition in which the number of atoms in the excited state is greater than the number of atoms in the ground state.

i.e., $\boxed{N_2 > N_1}$

To get population inversion, we need pumping and metastable state.

ii) Metastable state :(MSS) The energy states in which atoms can remain for unusually longer time (10^{-2} - 10^{-3} sec) compared to the other excited states (Life time is very short - $\simeq 10^{-8}$ sec) are called metastable states (MSS). They help to achieve inverted population in a system more easily.

Mechanism of Laser action : Three energy level system.



Consider a sample whose atoms can exist in three states (i.e., ground, excited & metastable state) as shown in fig.

Atoms in the ground state are pumped to state E_3 by a photon of energy $h\nu$ (E_3 - E_1). This state is unstable whose life time is about 10^{-8} sec, so that excited atoms undergo non-radiative transitions and go to energy level E_2. This energy level E_2 is called metastable state (MSS). The atoms stay in this metastable state for a considerable time (about 10^{-3} s). Thus the number of atoms in the excited state is more than that in the ground state. Thus the population has been achieved.

The atoms in the metastable state E_2 are now bombarded with photons of energy $h\nu$ ($\approx E_2 - E_1$). This results in stimulated emission gives rise to interse cohere-nt beam (Laser beam).

Classification of Lasers:

1} Solid state Lasers: Ex:- Ruby laser, Neodymium lasers.

2} Gas lasers: Ex:- He-Ne laser, Ion Lasers, CO_2 Laser etc.

3} Semi conductor Laser: Ex:- GaAS Laser, C^3 Laser.

4} Liquid, Dye & chemical laser: Ex:- Hcl Laser, HF Laser.

Carbon dioxide [CO_2] Laser

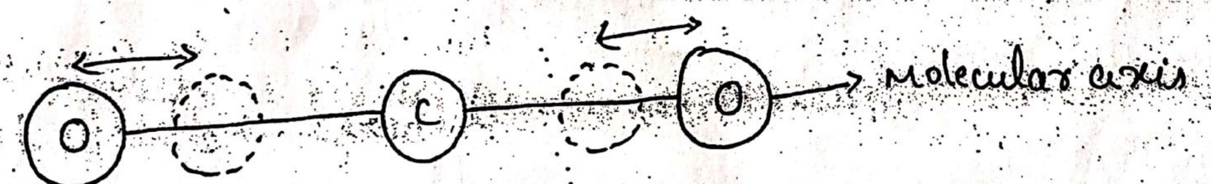
* CO_2 Laser was invented by an Indian Engineer C.K.N patel in 1963.
* It is a molecular gas laser which operates in the middle I-R region.
* The molecules posses vibrational and rotational Energies which are Quantized.

Fundamental modes of vibration in CO_2 molecule

In CO_2 molecule, there are 3 fundamental modes of vibration.

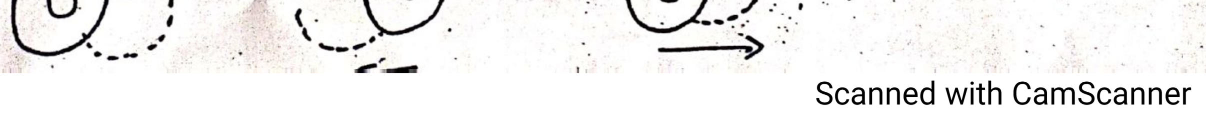
1) **Symmetrical streching mode :-**

In this mode, the carbon atom is stationary and the 2 oxygen atoms either simultainously move towards (or) away from the carbon atom (to & fro) along molecular axis.



2) **Asymmetric streching mode :-**

In this mode, both oxygen atom move in one direction while carbon atom moves in opposite direction along the molecular axis



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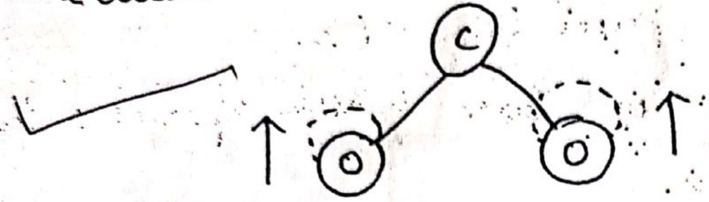
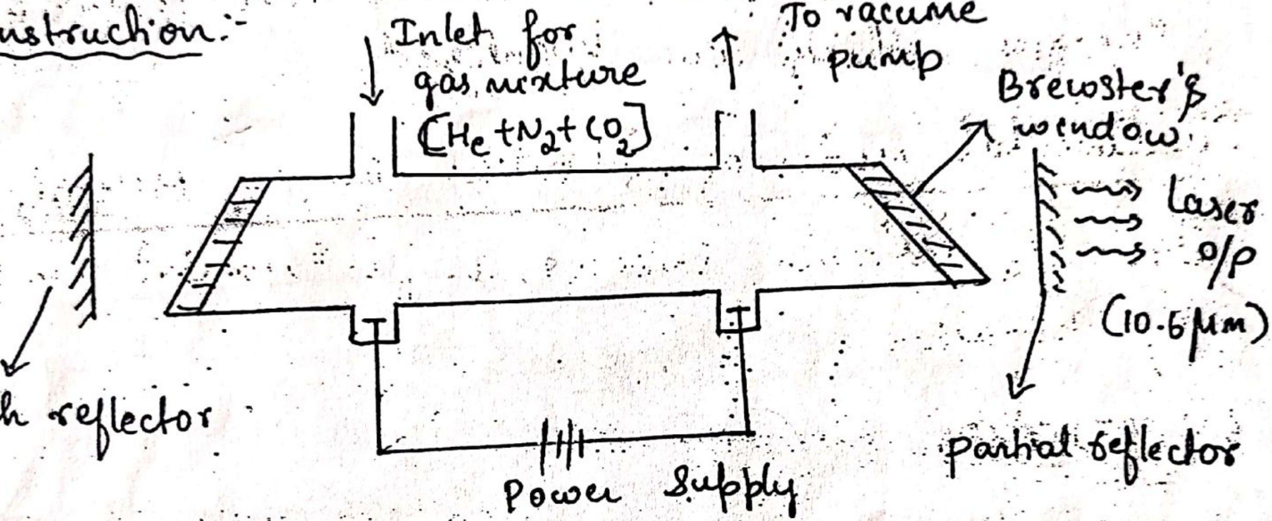
3) Bending mode:

In this mode; the oxygen atoms and a Carbon atom move perpendicularly to the molecular axis



Construction:



The CO_2 laser device consiste of very narrow glass tube of length 5cm & diameter 2.5cm the gas mixture consist of CO_2, Nitrogen & helium. the ratio of $CO_2 : N_2$ is 0.8 : 1 & more He atoms than N_2 in the glasstube. The pumping is by Electrical discharge. N_2 helps to increase the inverted popullation in CO_2 and Helium helps to depopullate the lower levels. The continious laser beam of wavelength 10.6 μm is transmitted through the Partially reflecting mirror.

The dissociation products like CO & O is removed with the help of vacume pump.

Working

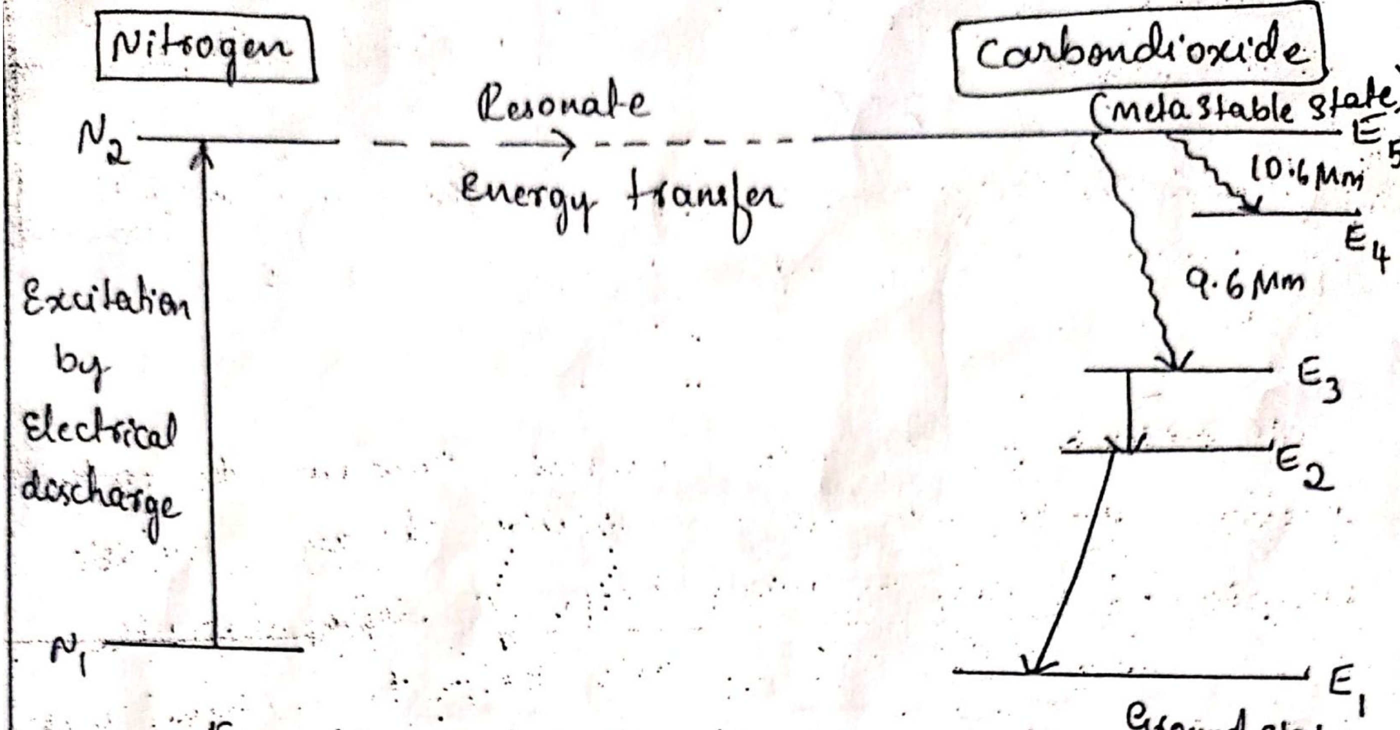


Fig Energy level diagram

When electric discharge takes place in the gas mixture both N_2 & CO_2 atoms absorb Energy and are excited to the higher Energy level. this Energy level matches with one of the vibrational- rotational level of CO_2 by E_5 shown in fig. Therefore more CO_2 atoms are raised to level E_5 by colliding with N_2 Molecules. As a result, there is resonance transfer of Energy from N_2 Molecule to the CO_2 molecule. This kind of Energy transfer is called resonate energy transfer

Now population inversion is Established between the E_5 level with respect to the 2 lower Energy levels E_4 & E_3, then these 2 possible transition takes place they are

(i) The transition from E_5 to E_4 level producess a radiation of wavelength 10.6 μm which is in IR region.

(ii) Transition from E_5 to E_3 level which give rise to radiation of wavelength 9.6 μm which is in I-R region.

The Exited CO_2 molecule transmit downwards to the ground state by non-radiative decays inelastic collision. The helium atoms helps to depopulates the lower Energy level in CO_2 and helps to conduct heat away to the walls, keeping CO_2 cooled. Also glass tube is made very narrow sothat CO_2 molecule may collide with the walls and transit to the Ground state, so that the lowest levels can be depopulated easily.

CO_2 laser can be used to find Extensive industrial applications such as welding, cutting drilling etc

Note:- He-Ne Laser has an Efficiancy of 0.01 to 0.1%. where as CO_2 laser operates with an Efficiancy of up to 30%.

Characteristics of a Laser beam:

The extra-ordinary features of a laser beam can be understand with reference by using following characteristics

1} Directionality :- Laser light is highly directional & is highly collimated beam of light.

2} Monochromaticity:- A Laser light has high degree of monochromaticity.

3} Coherence :- The phase difference between any two points is same through out & hence laser has a high degree of coherence.

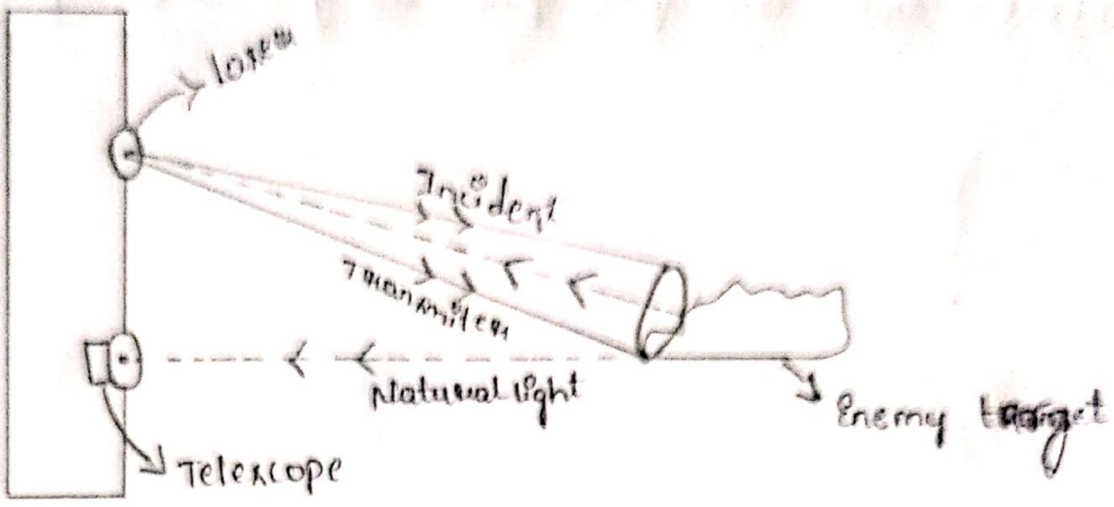
4} Light Intensity: Laser light is highly intensive, the intensity of laser light is hundred times better than the intensity of an ordinary light.

5} focussability: Since laser light is highly monochromatic and also highly collionated. It can be brought to a sharp focus by a lens.

Applications of lasers :-

Applications of laser :-

① Laser range finder in defense



The operation principle of a laser range-finder is the same as that of a conventional radar.

A ~~light~~ high powered pulsed laser beam from a solid state laser device such as Nd-YAG laser is directed on to the enemy target from a transmitt-er. upon incidence, the beam reflected from the surface of the target and recived as a signal by a reciver (echo). reciver consists of interferen-ce filter. The optical filter frequency is turned to the frequency of laser light. Therefore, all the background noise entering the receiver is wiped off. Finally the signals are amplified by using multiplier. The time taken by the signal is noted. The exact distance is measure (Distance = velocity x time)

The laser range finder is fitted with computers to provide information in a digital read out form. It also used for continuous tracking and ranging of missiles and aircrafts from the ground or from air

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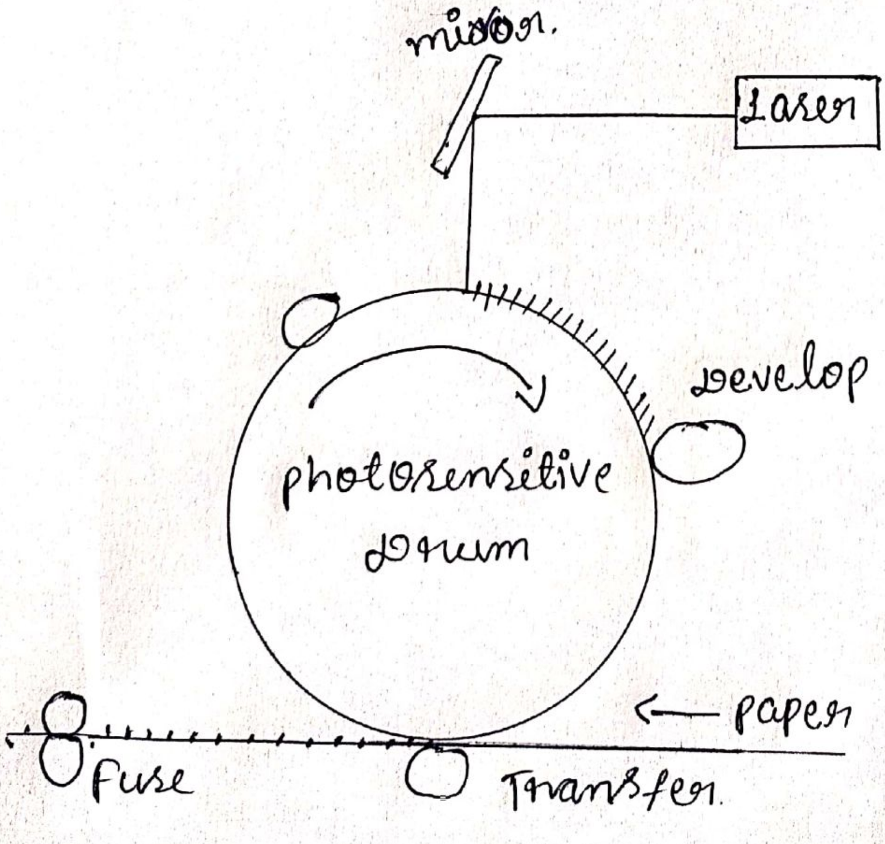
Laser printer:-

=> Laser printers were invented at XEROX in 1969 by Gary Starkweather. Laser printing is an Electro static digital printing process. It produces high Qulity text and Graphics by repeatedly passing a Laser beam back and forth over a negatively charged cylinder called a 'drum' to define a differentially Charged image. The drum then selectively collects electrically-charged powdered ink (toner) and transfers the image to paper, which is then heated to permanently fuse the text, image or both to the paper.

A diode Laser is used in the process of printing in laser printer



Advanges :-

1. Laser printer are generally quiet and fast
2. Laser printers can produce high quality output on ordinary papers
3. The cost per page of toner cartridges is lower than other printers

Disadvantages :-

1. The initial cost of Laser printers can be high
2. Laser printers are more expensive than dot-matrix printers and ink-jet printers.

Chandrashekar. K.S

MODULE - 03

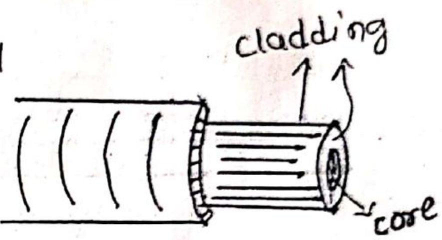
Optical Fibers

An optical fiber is a device which conduct light along an desired path without any appreciable Power loss.

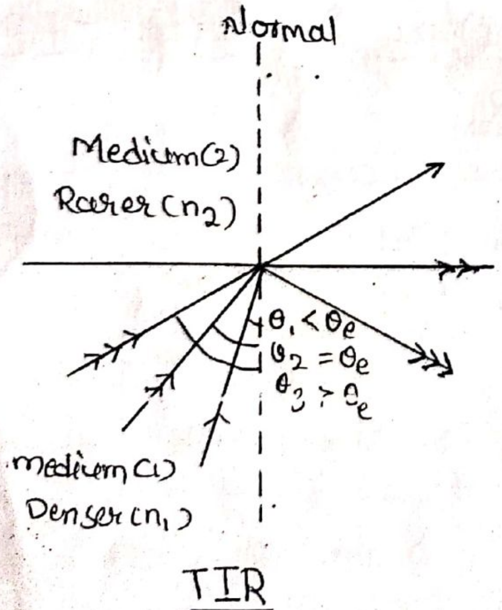
Optical fibers are used as a light guide (wave guide) in optical fiber communication.

* Construction:-

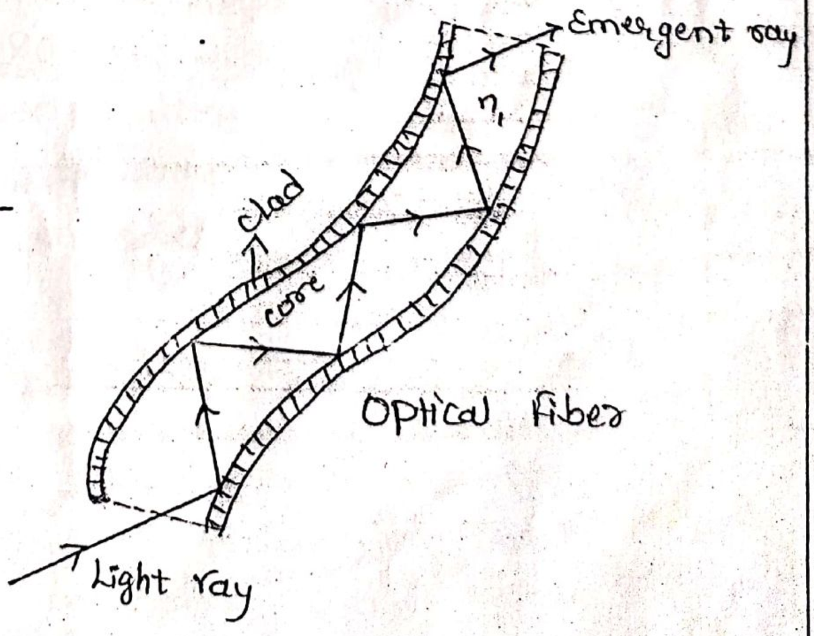
An optical fiber is in cylindrical shape as shown in the fig. It has 2 parts i.e., core & cladding. These parts are made by glass or plastics. The refracting index of the cladding is less than the core. The cladding is enclosed in a polyethane jacket which safeguards the fiber against chemical reaction with the surroundings.



Propogation mechanism or optical fibers work as wave-guides



TIR



Propagation Mechanism:-

The mechanism of light propagation in the optical fiber is based on the principle of total internal reflection (TIR) i.e., when light travels from a denser medium to a rarer medium, if the angle of incidence is greater than the critical angle, light gets totally reflected back into the denser medium without undergoing refraction.

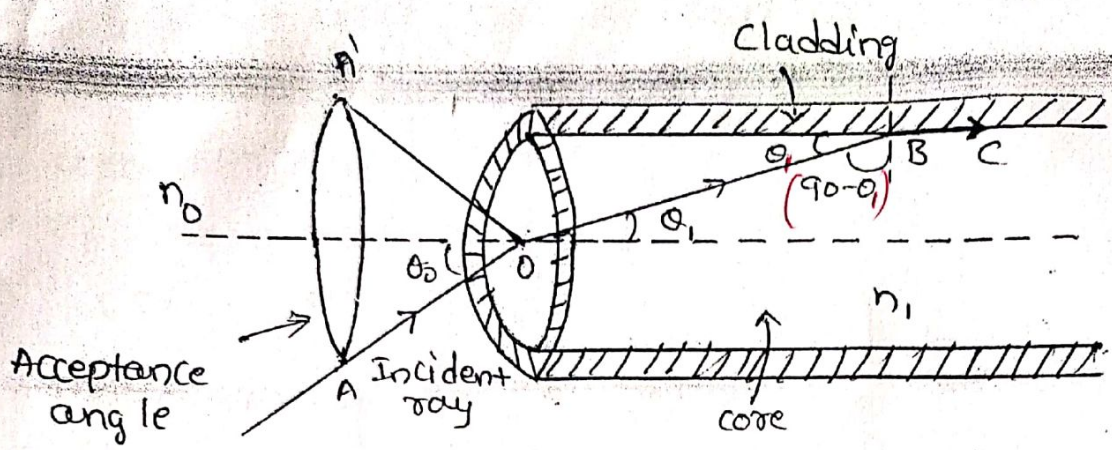
Optical fiber or wave guide is a thin long glass or transparent polymer fiber. It's central part has higher refractive index n_1 and is called the core. The core is surrounded by a refractive index n_2 and is called the clad $(n_1 > n_2)$. When a ray of light enters the core of the optical fiber through one end face, at a suitable angle, it falls on the core-clad interface and undergoes TIR. The reflected ray again falls on interface and interface TIR. Thus, the light ray undergoes a very large number of reflections and travels through the optical fiber.

Angle of Acceptance(α) And Numerical Aperture(NA)

OR

Q. Explain with a neat diagram of acceptance angle and numerical aperture of an optical fiber. Hence derive an expression for numerical aperture.

Defn: Angle of acceptance is the maximum value of the angle of incidence at one end face of an optical fiber, below which the ray entering the core propagation through the fiber by means of total internal reflection.



When a ray of light 'AO' is incident at 'O' at an angle 'θ_0'. This ray undergoes refraction at an angle 'θ_1'. In the core and further proceeds to fall at critical angle of incidence $[\approx(90-\theta_1)]$ at 'B' on the interface b/w core and cladding. The ray 'OB' grazes along 'BC'

It's clear that from the figure that any light ray that enters into the core at an angle of incidence less than θ_0 undergoes total internal reflection on the other hand, any light ray that enters at an angle of incidence greater than 'θ_0' at 'O'. Get refracted into the cladding region. The ray donote undergo that internal reflection

The angle 'θ_0' is called "Acceptance Angle". and '$\sin\theta_0$' is called "Numerical Aperture" (N.A). of the fiber. "The light gathering capacity of an optical fiber is known as "Numerical aperture."

Let n_0, n_1 & n_2 be the refractive index of surrounding medium, core and cladding respectively.

Applying snell's law at point 'o'.

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 \quad \text{--- (1)}$$

~~At~~ the point 'B' on the interface, the angle of incidence $= (90-\theta_1)$

Again applying Snell's law at point 'B'', we get

$$n_1 \sin(90-\theta_1) = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2 \sin 90$$

$$n_1 \cos\theta_1 = n_2$$

$$\left(\cos\theta_1 = \frac{n_2}{n_1}\right) \quad \text{—} (2)$$

Re-writing eqⁿ (1), we have

$$\sin\theta_0 = \frac{n_1}{n_0} \sin\theta_1 \qquad \left[\because \sin\theta_1 = \sqrt{1-\cos^2\theta_1}\right]$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{(1-\cos^2\theta_1)} \quad \text{—} (3)$$

Substitute eqⁿ (2) in eqⁿ (3) we get

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}} = \frac{\cancel{n_1}}{n_0}\frac{\sqrt{n_1^2-n_2^2}}{\cancel{n_1}}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2-n_2^2}}{n_0}$$

The surrounding medium of the fiber is air, then $n_0 = 1$

$$\therefore \sin\theta_0 = \sqrt{n_1^2-n_2^2} \qquad [\because \sin\theta_0 = N.A]$$

$$\text{i.e.,} \quad \left(N.A = \sqrt{n_1^2-n_2^2}\right)$$

Above Expression is numerical aperture.

If θ_i is the angle of incidence, then the ray will be able to propogate.

If $\theta_i < \theta_0$

If $\sin\theta_i < \sin\theta_0$ or

$$\sin\theta_i < \sqrt{n_1^2-n_2^2}$$

$$\text{i.e.,} \quad \boxed{\sin\theta_i < N.A}$$

This is the condition for propagation.

* Fractional Index Change (Δ):

It's "the ratio of refracting index difference b/w the core and cladding to the refractive index of core of the optical fiber".

$$\therefore \boxed{\Delta = \frac{(n_1 - n_2)}{n_1}}$$

where,

$n_1 \rightarrow$ Refractive Index of core.

$n_2 \rightarrow$ Refractive Index of cladding.

Relation b/w N.A and fractional Index Change (Δ)

W.K.T

$$\Delta = \frac{(n_1 - n_2)}{n_1}$$

$$n_1 \Delta = (n_1 - n_2) \quad \text{---} (1)$$

we have,

$$a^2 - b^2 = (a+b)(a-b)$$

$$N.A = \sqrt{n_1^2 - n_2^2} \qquad [\because n_1^2 - n_2^2 = (n_1+n_2)(n_1-n_2)]$$

$$= \sqrt{(n_1+n_2)(n_1-n_2)}$$

$$N.A = \sqrt{(n_1+n_2)\Delta n_1} \qquad \text{from eq}^n (1)$$

If $n_1 = n_2$, $(n_1 + n_2) = 2n_1$

$$N.A = \sqrt{2n_1 \cdot \Delta n_1}$$

$$N.A = \sqrt{2n_1^2 \cdot \Delta}$$

or $\therefore \boxed{N.A = n_1\sqrt{2\Delta}}$

This shows that as Δ increases, numerical aperture (N.A) also increases.

* V-number:-

"The number of modes supported for light propagation in an optical fiber is determine by a parameter called V-number". (denote as V)

The V-number is given by

$$V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2}$$

or

$$\boxed{V = \frac{\pi d}{\lambda} (N.A)}$$

Where,

d → diameter of the core

λ → wavelength of light propogating in the fiber

n_1 → Refractive index of core

n_2 → Refractive index of cladding.

If the fiber is surrounded by a medium of refractive index n_0, then

$$V = \frac{\pi d}{\lambda} \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

For $V \gg 1$, the number of modes supported by the fiber is given by,

Number of modes, $\boxed{m \simeq \frac{V^2}{2}}$

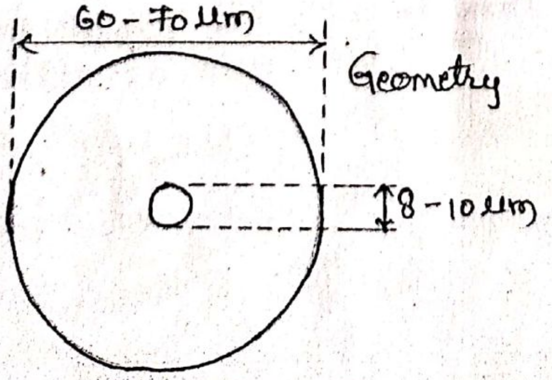
Types Of Optical Fiber

The optical fibers are classified into 3 catagories, namely

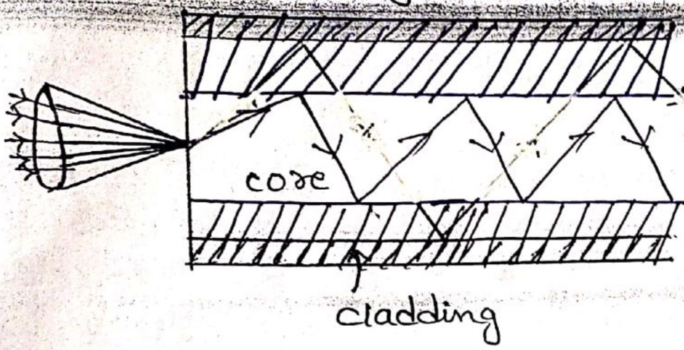
1) Single mode fiber
2) step index multimode fiber
3) Graded index multimode fiber.

The classification is done depending on the refractive index profile and the number that the fiber can guide.

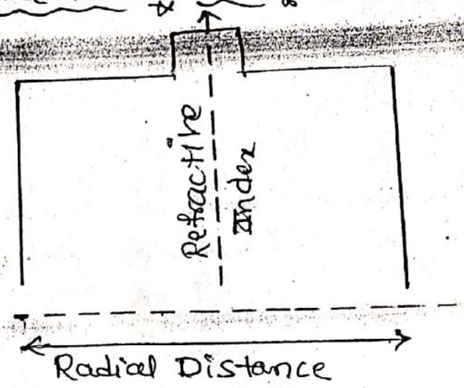
1) Single mode fiber



Ray Propagation



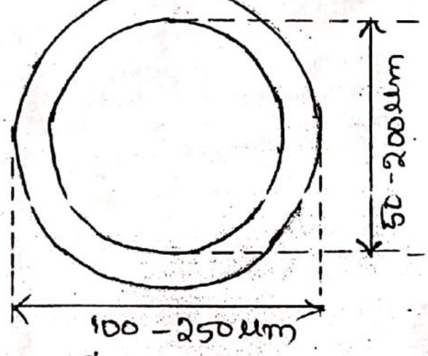
Refractive Index Profile



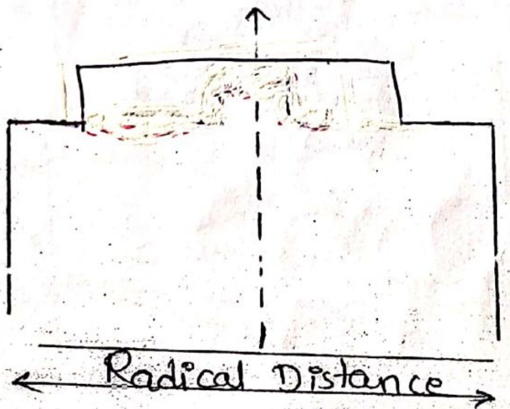
The Geometry, R.I profile & ray propogation in Single mode fiber is as shown in figure.

It consists of a core and a cladding. The refractive index of core material is greater than that of cladding material. Since the diameter of core is very less, it can able to support just single mode, hence it's called single mode fiber. It requires laser as source of light.

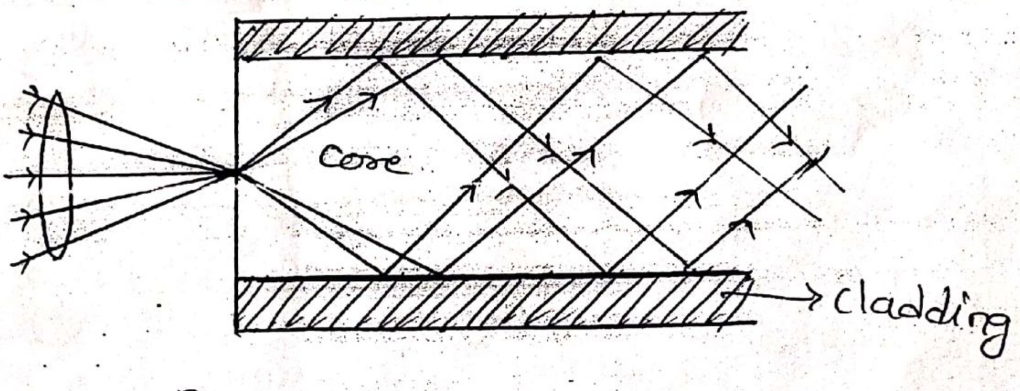
2) Step-Index multimode fiber.



Geometry



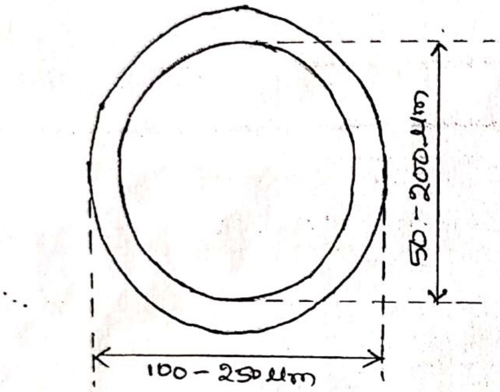
Refractive Index Profile.



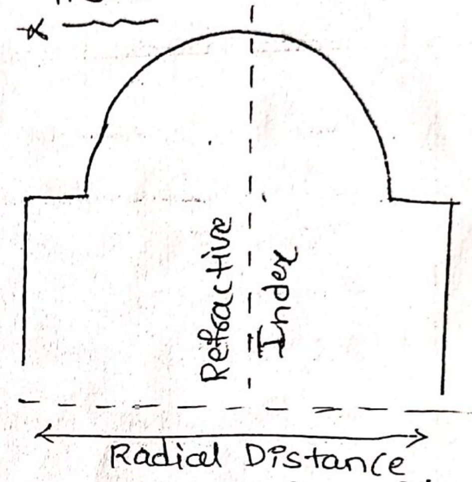
Ray Propagation

The geometry, R.I profile & ray propagation in a step-index multimode fiber is as shown in above figure It's similar to that of single mode fiber, But the difference is that the diameter of core is higher than the single mode fiber. so that it can support large number of modes and it's R.I profile represent step function hence it's called step index multimode. It requires LED or Laser as source of light.

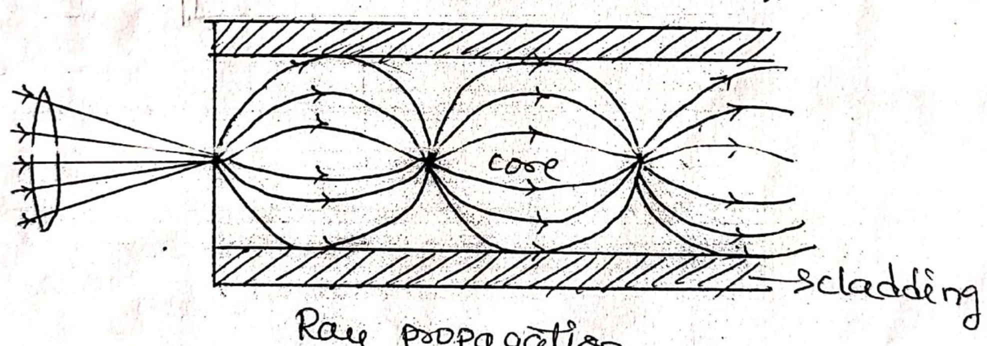
3> Graded Index Multimode fiber



Geometry



Refractive Index of Profile



Ray propagation

The Geometry, R.I, Profile & Ray propagation in graded index multimode fiber is as shown in above figure.

The geometry of the graded index multimode fiber is same as that of step index multimode fibre But it's core refractive index value decreases in radially outward direction from the axis & becomes equal to that of the cladding at the interface But refractive index of the cladding is same. It requires LED or Laser as source of light.

* Attenuation or Power loss or fiber loss

"Attenuation is the loss of power of the optical signal as it propagates through the fiber".

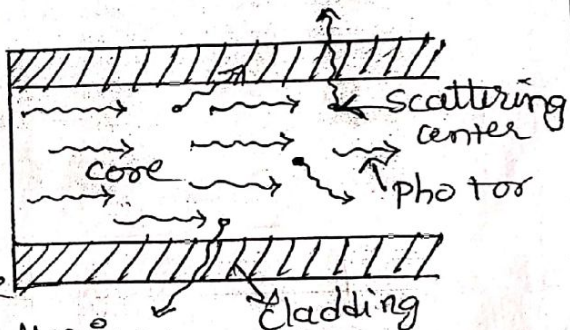
There are three types of attenuation namely

1. Absorption loss
2. Scattering loss
3. Radiation loss.

1. Absorption loss:

When signal propagates through the fiber a few photons associated with the signal are absorbed by impurities present in the fiber. This results in power loss.

2. Scattering loss:



When a signal propagates through the fiber a few photons associated with the signal are scattered by the scattering objects such as impurities present in the fiber. The dimensions of the scattering objects are very small compared to the wavelength light. This type of scattering is similar to 'Rayleigh scattering'. It's found that the co-efficient of scattering is inversely proportional to the wave-length of the object.

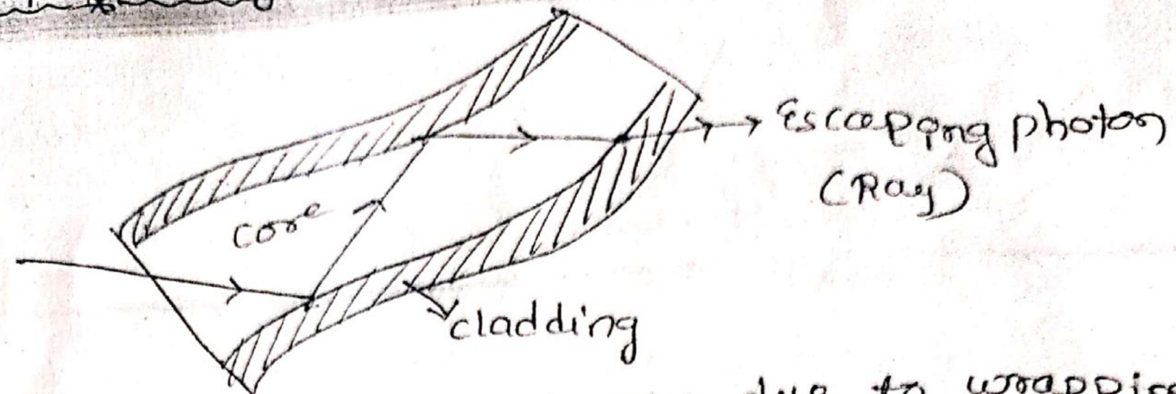
3. Radiation losses:

It is due to the bending of fibers and they are two types of bends.

a) Macroscopic bends

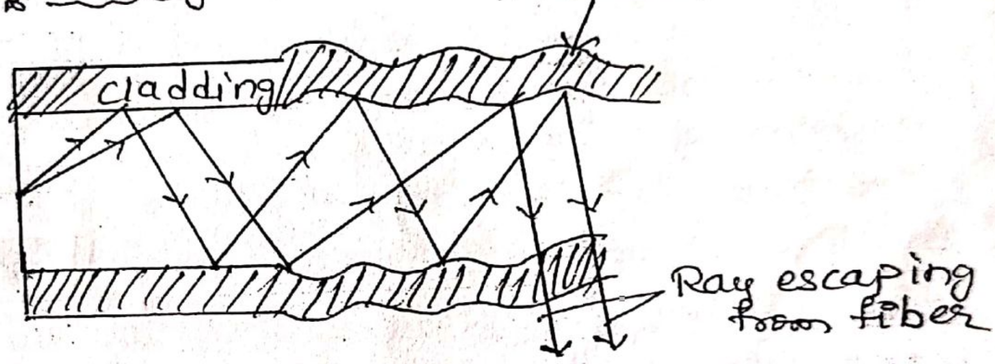
b) Microscopic bends

a) Macroscopic bendings:



This type of bending occurs due to wrapping of fibers while manufacturing. If the bending is too sharp, because of this some of modes escape from fiber that result in very high power loss.

b) Microscopic Bending:



This type of bending is due to the non uniformity in the fiber while manufacturing because of their a few modes undergo leakage. this result in power loss.

* Attenuation co-efficient (α)

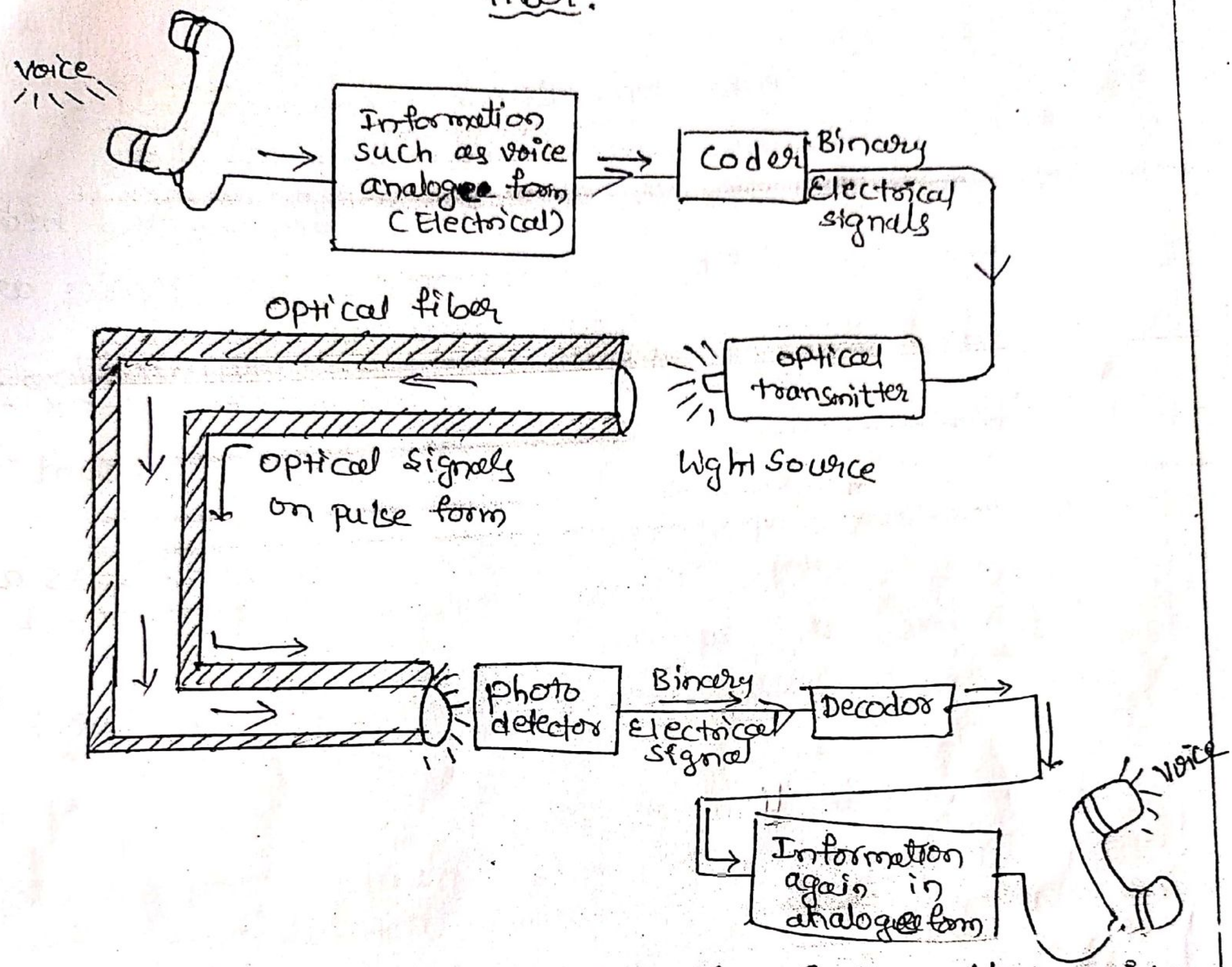
$$\boxed{\alpha = -\frac{10}{L}\log_{10}\left(\frac{P_{out}}{P_{in}}\right)} \ dB/Km$$

where L = length of the fiber

α = attenuation co-efficient

APPLICATIONS OF OPTICAL FIBERS

1) Point to Point Communication System using optical fiber:



In point-to point communication system the voice in the analogue form(signal) is fed to coder which convert analogue signal to binary signal. The binary signal applied to the optical transmitter. The optical transmitter converts electrical signal into optical light signal. the light signal propagates through optical fiber and reaches the photo detector. The photodetector converts light signal to the electrical signals. The electrical signal is converted into analogue signal by using decodor. Finally the analogue signal is fed to the telephone receiver to hear sound.

Fibre optic sensors

⇒ A device that uses light guided within an optical fibre for detection of an external physical, chemical or biomedical parameter is called a fibre optic sensor (FOS)

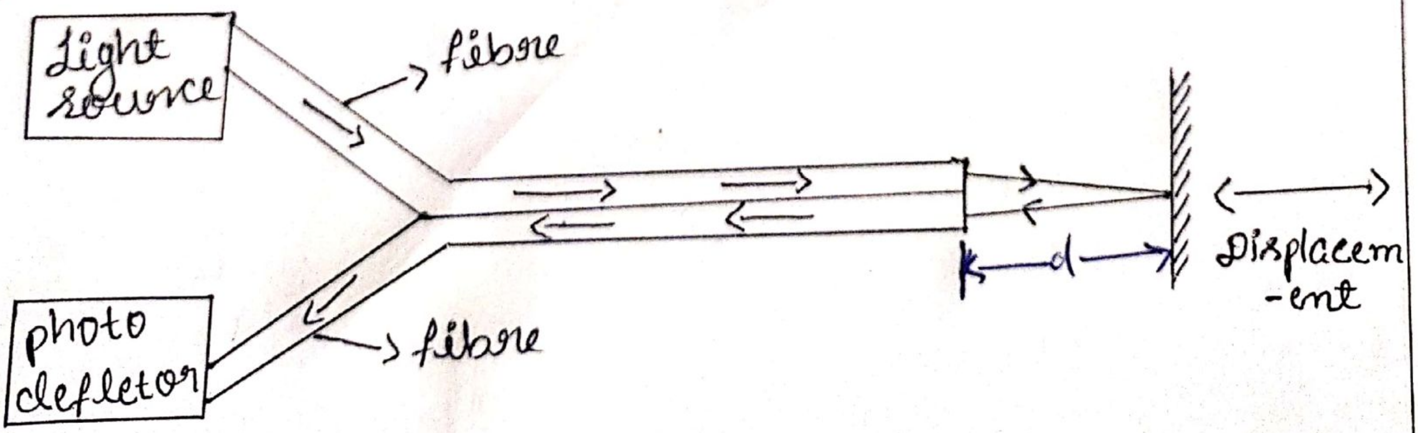
① Displacement sensors

Principle

⇒ The basic principle employed in displacement sensor consists of using adjacent pair of optical fibre, one to carry light from a source to an object whose displacement or motion is to be measured and the other to recieve the light reflected from the object and carry it back to a photodefector.

Construction:

⇒ Two separate optical fibres are placed adjacent to each other. on of them transmits light coming from a light source. The other fibre recieves light reflected from the object under study and passes it into a photodeflector.



working

⇒ Light from the source passes through one optical fibre and incident on the target. The reflected light reaches the photodetector through another optical fibre. Light reflected from target and collected by the detector is a function of the displacement of the target may be registered at the optical detector

By proper calibration, we can obtain the displacement of the object interms of the stren -gth of the output signal of the photodetector.

Advantages of optical fiber used communication System

1> Optical fibers can able carry large amount of information.

2> Materials used for preparing optical fibers are Plastic & silicon dioxide. Because of this cost becomes very low.

3> Optical fiber having life time is very high about 40 yrs.

4> They are light weight.

5> Because of superior attenuation characteristics optical fibers can transmitts signals effectively over a long distance

6> There is no lightening or sparkling

7> There is no interference b/w on communication channel to the other

* Disadvantages:

1> Splicing is a complicated task in optical fibers. It's not done properly the power loss becomes very high.

2> Under certain circumstances the fibers may suffer line break. The re-establishment of connections is very much complicated and time consuming.

Module-1

Wave - Particle Dualism.

Phenomena such as photoelectric effect and compton effect give evidence that the light behaves like a "Particle nature". But the phenomena like interference, diffraction and polarisation can be explained on the basis of "Wave nature".

Hence light behaves like particle and wave nature.

de-Broglie concept of matter wave (or de-Broglie's hypothesis)

In 1924 de-Broglie made a hypothesis that moving particle sometimes behaves like wave as well as particle. According to de-Broglie, electrons just like light have dual nature.

His suggestion was based on two assumptions. They are

(i) The Universe is made of particles and radiation and both these entities must be symmetrical.

(ii) The nature loves symmetry.

The waves associated with the moving particle or matter are known as "de-Broglie waves" or "matter waves".

de - Broglie wavelength.

Consider a particle of mass 'm' moving with a velocity 'V'.

According to Einstein's Eqn $E = mc^2 \longrightarrow ①$

According to Planck's hypothesis $E = h\nu = \frac{hc}{\lambda} \longrightarrow ②$

From ① & ②

$$m\cancel{c}^{2} = \frac{h\cancel{c}}{\lambda}$$

$$\boxed{\lambda = \frac{h}{mc}}$$

But particle moves with velocity V, then $\boxed{\lambda = \frac{h}{mV}}$ or $\boxed{\lambda = \frac{h}{P}} \longrightarrow ①$

where, P = momentum

Consider a particle of mass 'm' moving with a velocity 'V'.
Then the kinetic energy E_K of the particle is given by

$$E_K = \frac{1}{2} mV^2$$

$$2E_K = mV^2$$

$$V^2 = \frac{2E_K}{m}$$

$$V = \sqrt{\frac{2E_K}{m}} \longrightarrow ②$$

Substitute ② in ①

$$\therefore \lambda = \frac{h}{m\sqrt{\frac{2E_K}{m}}} = \frac{h}{\sqrt{\frac{m^{\cancel{2}}2E_K}{\cancel{m}}}}$$

$$\therefore \boxed{\lambda = \frac{h}{\sqrt{2E_K m}}}.$$

∴ de - Broglie wavelength associated with electrons

$$\lambda = \frac{h}{\sqrt{2meV}} \qquad (\because E_K = eV)$$

or

$$\lambda = \frac{1}{\sqrt{V}} \left(\frac{h}{\sqrt{2me}} \right) \longrightarrow (3)$$

By substituting the values of ;

$h = 6.625 \times 10^{-34}$ JS

$m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ C

then eqⁿ (3) $\Rightarrow$ $\lambda = \frac{1}{\sqrt{V}} \left[\frac{6.625 \times 10^{-34} \text{ JS}}{\sqrt{2 \times (9.1 \times 10^{-31} \text{ kg}) \times (1.6 \times 10^{-19} \text{C})}} \right]$

$$\lambda = \frac{1.226 \times 10^{-9}}{\sqrt{V}} \text{ m.}$$

or

$$\boxed{\lambda = \frac{1.226}{\sqrt{V}} \text{ n.m}} \longrightarrow (4)$$

Characteristics of matter waves (or) De-Broglie Waves:

① A Particle of mass 'm' moving with velocity 'v' then the wavelength of wave associated with the particle is

$\lambda = \frac{h}{mv}$ or $\lambda = \frac{h}{p}$.

② Matter wave is different from electromagnetic waves.

③ The matter wave associated with a particle is independent to the charge of that particle.

④ The velocity of matter wave is not constant like the electromagnetic radiation, but it dependents on the velocity of particle associated with it.

⑤ The velocity of matter wave is normally greater than the velocity of light.

⑥ If the mass of particle is small, The wavelength associated with that particle is greater and vise versa.

⑦ If velocity of particle is zero then wavelength will be indeterminate. If $V = \infty$, Then $\lambda = 0$. This means that the matter waves are generated by motion of the particle

Phase velocity (v_p)

The state of vibration of a particle is called phase. The velocity with which the wave propagating with constant phase is known as "Phase velocity"

$$\boxed{V_p = \frac{\omega}{K}}$$

where $\omega \rightarrow$ Angular frequency
$K \rightarrow$ wave number

Group velocity (v_g).

It is the velocity with which the wave packet is formed due to the superposition of two (or) more waves of slightly different wavelengths travelling along the same direction of propagation is called "group velocity"

(or)

The velocity with which the wave packet moves is called "group velocity"

$$\boxed{V_g = \frac{d\omega}{dK}}$$

Quantum Mechanics

Quantum Mechanics is the branch of physics that deals with the motion of particles on the atomic or subatomic scale.

classical mechanics is an approximation of Quantum mechanics. The term 'Quantum' mechanics was coined by "Max Born" in 1924.

A particle occupies a definite place in space & will have definite momentum. This happens in classical mechanics. But in Quantum mechanics (in atomic scale) it is not possible to locate position & momentum of a particle simultaneously.

the uncertainity principle states that "it is impossible to determine simultaneously both the position & momentum of a particle with accuracy".

then, it was Heisenberg, who first realised the existance of the reciprocal relation between the uncertainities of certain pairs of physical quantities like position - momentum pair. Energy-time pair & angular momentum - angular position.

Statement of Heisenberg Uncertainity Principle:

①. "In any simultaneous measurement of position & momentum of a particle, the product of corresponding uncertainities inherently present in the measurement is equal to (or) greater than $\frac{h}{4\pi}$".

i.e $\boxed{\Delta x . \Delta p \geq \frac{h}{4\pi}}$

②. "In any simultaneous measurement of energy & time in a physical process, the product of the corresponding uncertainities inherently present in the measurement is equal to (or) greater than $\frac{h}{4\pi}$".

i.e $\boxed{\Delta E . \Delta t \geq \frac{h}{4\pi}}$

③. "In any simultaneous measurement of angular displacement (θ) and angular momentum (L) in a physical process, the product of the corresponding uncertainities inherently present in the measurement is equal to (or) greater than $\frac{h}{4\pi}$".

i.e $\boxed{\Delta L \cdot \Delta\theta \geq \frac{h}{4\pi}}$

Physical Significance of Heisenberg's uncertainity principle:

The physical significance of uncertainity principle is that we cannot think of exact position (or) an accurate value for momentum of a particle. Instead of one should think of probable momentum of the particle. The estimation of such probabilities are made by means of certain mathematical functions. such as Probability function. In Quantum mechanics similar interpretation is made for the conjugate pairs like ΔE & Δt, ΔL & $\Delta\theta$.

Application of Uncertainity Principle : [8M]

[Non existance of electron in the nucleus]:

The uncertainity principle has been successfully in explaining many phenomenon observed in nature. Among them the important application is proving the non-existance of electron inside the nucleus.

We know the diameter of nucleus is of the order of 10^{-14} m. If an electron is to exist inside the nucleus, then the uncertainity in its position 'Δx' must not exceed the size of the nucleus.

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i.e $\Delta x \leq 10^{-14}$ m.

∴ uncertainity in momentum is,

$$\Delta x . \Delta P \geq \frac{h}{4\pi}$$

$$\Delta P \geq \frac{h}{4\pi . \Delta x}$$

$$\geq \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 10^{-14}}$$

$$\boxed{\Delta P \geq 0.5 \times 10^{-20} Ns}$$

∴ The momentum of the electron must be atleast equal to the above value i.e $\Delta P \geq 0.5 \times 10^{-20}$ Ns.

In order to make electron exist within the nucleus its energy 'E' equation is given by

$$E_{min} \geq \frac{P^2}{2m}$$

But $\Delta P \simeq P$, $m = 9.1 \times 10^{-31}$ kg (mass of electron)

$$\therefore E_{min} \geq \frac{(0.5 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}}$$

$$E_{min} \geq 1.372 \times 10^{-11} J$$

In terms of 'eV',

$$E_{min} \geq \frac{1.372 \times 10^{-11}}{1.6 \times 10^{-19}}$$

$$\geq 85.75 \times 10^{6}$$

$$\therefore \boxed{E_{min} \geq 85.75 MeV}$$

Thus the free electron to exist inside a nucleus must have a minimum energy is greater than or equal to 85.75 MeV. But experimental evidence shows that maximum energy (K.E) of β - particle (electrons) emitted from the nucleus is of order 4MeV. This clearly indicates that electrons cannot exist inside the nucleus.

Wave function:

The wave function describes the physical situation of the wave associated with a particle. It contains all the information about the system. The wave function is denoted by Greek letter ψ (Psi). $\psi = \psi(x, y, z, t)$. ψ is generally complex with both real & imaginary parts.

The wave function which is obtained by solving the fundamental equation called "**Schrodinger Equation**".

The Time-dependent Schrodinger equation is,

$$\frac{-h^2}{8\pi^2 m} \frac{d^2\psi}{dx^2} + V\psi = \frac{-ih}{2\pi} \frac{d\psi}{dt}$$

The Time-Independent Schrodinger equation is,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0.$$

Properties and Physical Significance:

The most important property of the wave function 'ψ', is that it gives measure of finding the particle at a particular position (x, y, z) & at time 't'. i.e wave function 'ψ' enables all possible information about the particle has to be derived.

The following are the basic properties of the wave functions are,

1). The wave function has no direct physical significance. i.e, it can interfere itself (phenomenon of diffraction).

2). It is large in magnitude where the particle such as electron (or) photon is likely to be located & small at other places.

3). The wave function discribes the behaviour of a single particle (or) photon & not the statistical distribution of a number of particle (or) quanta.

Physical significance:- of wave function $\psi(x,t)$ is the solution of Schrodinger wave equation.

*. It gives quantum mechanically complete discription of the behaviour of moving particle.

*. The wave function ψ cannot be measured directly by any physical experiment. But $|\psi|^2$ gives probability of finding the particle in a elemental space. $d\tau$ of volume τ, Hence $|\psi|^2$ is called probability function (or) probability density.

① Probability density:

The probability of finding the particle described by the wave function ('ψ') is proportional to $|\psi|^2$. The $|\psi|^2$ [square of the magnitude - amplitude] is called probability density.

Let the particle be present inside the volume 'V', but where exactly the particle is situated inside the volume is not known. Then the probability of finding the particle in a certain element of volume 'dv' is given by $|\psi|^2 . dv$. $\therefore$ $|\psi|^2$ is called probability density (or) probability function.

Explain why the basic function 'ψ' has no physical significance:

The probability of occurance of an event is real & positive. But the wave functions are complex. Hence ψ has no physical significance.

$\psi\psi^*$ has got Physical significance:

In order to get the value i.e in positive & real, the wave function ψ is multiplied with its complex conjugate ψ^*.

The product of $\psi\psi^*$ is always real & positive quantity.

$\therefore$ probability density is given by $|\psi|^2 = \psi^* . \psi$

(2) Normalization:

The probability of finding the particle in certain volume element 'dv' is $|\psi|^2$ dv. If the particle exists somewhere in space (or) bounded region, then the total probability must be unity. But limit extends from $-\infty$ to ∞. Thus

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 . dv = 1} \quad \text{(or)} \quad \boxed{\int_{-\infty}^{\infty} |\psi|^2 . d\tau = 1} \quad \text{if } dv = d\tau$$

A wave function which satisfies above equation is said to be "**Normalized**". Normalization helps us to calculate the constant present in ψ.

Time independent one-dimension schrodinger wave equation

The wave function describing the de-broglie wave along positive x- direction can be written in complex notation as

$$\Psi = Ae^{i[kx-\omega t]} \longrightarrow (1)$$

where Ψ is total wave function. 'A' is constant, ω is angular frequency & $i=\sqrt{-1}$

The time independent part of the equation (1) is

$$\psi = Ae^{ikx}$$ where ψ = time independent wave function.

$\therefore$ Eq^n (1) becomes,

$$\Psi = Ae^{ikx} . e^{-i\omega t}$$

$$\Psi = \psi e^{-i\omega t} \longrightarrow (2)$$

Differentiate eq^n (2) twice w.r.t 'x', we get.

$$\frac{d^2\Psi}{dx^2} = e^{-i\omega t} \frac{d^2\psi}{dx^2} \longrightarrow (3)$$

Differentiate eq^n (2) twice w.r.t 't' we get

$$\frac{d\Psi}{dt} = (-i)(\omega)\psi e^{-i\omega t}$$

$$\frac{d^2\Psi}{dt^2} = (-i)(-i)\omega^2 \psi e^{-i\omega t}$$

$$\frac{d^2\Psi}{dt^2} = i^2 . \omega^2 \psi e^{-i\omega t}$$

$$\frac{d^2\Psi}{dt^2} = -\omega^2 \psi e^{-i\omega t} \longrightarrow (4) \qquad (i^2=-1)$$

The wave function for a de-Broglie wave in second order differential form in one dimension is written as.

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \rightarrow (5)$$

where v = wave velocity

Substituting eqⁿ (3) & eqⁿ (4) in eqⁿ (5)

$$e^{-i\omega t}\frac{d^2\psi}{dx^2} = \frac{1}{v^2}(-\omega^2\psi e^{-i\omega t})$$

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v^2}\psi \rightarrow (6)$$

But $\omega = 2\pi\nu$

$$\omega = 2\pi\left(\frac{V}{\lambda}\right) \qquad (\because \nu = V/\lambda)$$

$$\frac{\omega}{V} = \frac{2\pi}{\lambda}$$

$$\frac{\omega^2}{V^2} = \frac{4\pi^2}{\lambda^2} \rightarrow (a)$$

Substituting eqⁿ (a) in eqⁿ (6)

$\therefore$ eqⁿ (6) becomes,

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \rightarrow (7)$$

$$\left[\because \lambda^2 = \frac{h^2}{m^2v^2}\right]$$

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∴ Total Energy (E) of the particle is given by,

$$\text{Total Energy} = K.E + P.E$$

$$E = \frac{1}{2} mv^2 + V$$ where $v \rightarrow$ wave velocity

$V \rightarrow$ potential energy.

$$(E - V) = \frac{1}{2} m v^2$$

$$2(E - V) = mv^2$$

$\times^{ly}$ 'm' on b.s,

$$2m(E - V) = m^2 v^2 \longrightarrow (8)$$

∴ eq^n (7) becomes,

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 2m(E-V)\psi}{h^2} = 0$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0} \longrightarrow (9)$$

The eq^n (9) gives time - independent one - dimension schrodinger's wave equation.

For a free particle. Potential Energy $V = 0$, then eq^n (9) becomes,

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0} \longrightarrow (10)$$

eq^n (10) gives time - independent schrodinger equation for a free particle.

Eigen functions and Eigen values:

The schrodinger wave equation is a second order differential equation. Hence it has many solutions for ψ. But only few wave functions are acceptable. If acceptable wave function should satisfies the following conditions.

i.e ①. ψ must be single valued every where.

②. ψ must be finite every where.

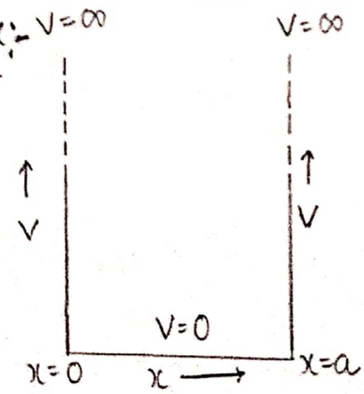
③. ψ & its derivative must be continous every where.

Such acceptable wave functions are called "**Eigen functions**".

The Eigen functions are used in schrodinger's equation to evaluate energy (E). Since eigen functions are restricted to finite and the corresponding energy value (E) are also finite. The energy values are called "**Eigen values**".

Applications of Schrodinger wave Equation: [6M]

* Particle in one dimensional potential well of infinite depth (Or) particle in a box:-



$v(x) = 0 \quad 0 \leq x \leq a$

$v(x) = \infty \quad x < 0, \; x > a$

Consider a particle of mass 'm' is free to move in the x-direction only in the region $x=0$ to $x=a$. Hence potential of the particle with in the box (i.e $0 \leq x \leq a$) is zero. ∴ The particle bounded within the limits $x=0$ & $x=a$. This type of configuration of potential in space is called "infinite potential well". A particle bound with in such an infinite potential is called "Particle in box". outside the well. The potential is taken as infinity.

The Schrodinger's eq^n for outside the well is given by,

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-\infty)\psi = 0 \longrightarrow (1) \quad [\because V=\infty]$$

This eq^n implies that the electron cannot found outside the box. The wave function (ψ) is zero every where outside the box.

$\therefore$ Schrodinger's eq^n for the particle inside the well is given by

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \longrightarrow (2) \quad [\because V=0]$$

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \longrightarrow (3) \quad \text{where } K^2 = \frac{8\pi^2 mE}{h^2}$$

The solution for the above eq^n (3) is given by

$$\boxed{\psi = A\cos kx + B\sin kx} \longrightarrow (4)$$

where A & B are two constant values, which are determined from the boundary conditions.

(a). condition 1: If $\psi = 0$ at $x = 0$

$\therefore$ Eq^n (4) becomes,

$$0 = A\cos(0) + B\sin(0)$$

$$0 = A(1)$$

$$\boxed{A = 0}$$

(b). condition (2) : If $\psi = 0$ at $x = a$

$\therefore$ Eq^n (4) becomes, $\Rightarrow 0 = A\cos ka + B\sin ka$

But $A = 0$ & $B \neq 0$

$0 = 0 + B\sin ka$

$\therefore \boxed{\sin ka = 0}$

$ka = \sin^{-1}(0)$

$ka = n\pi \Leftarrow$ ($\because \sin n\pi = 0$) or ($\sin^{-1}(0) = n\pi$)

$\boxed{K = \frac{n\pi}{a}}$ $(n = 0, 1, 2, 3, \ldots\ldots)$

Substitute the value of A & K in eq^n (4), we get,

$$\psi = 0 + B\sin\left(\frac{n\pi}{a}\right)x.$$

$$\boxed{\psi_n = B\sin\left(\frac{n\pi}{a}\right)x} \longrightarrow (5)$$

This represents the permitted solutions (or) Eigen functions. To evaluate 'B' in eq^n (5) one has to perform the normalization of the wave function.

Normalization (or) (Evaluate B):

The integral of the square of the wave function over the entire space in the well must be equal to unity because, there is only one particle & at any time it is present some where inside the well only.

$$\therefore \int_0^a |\psi_n|^2 . dx = 1.$$

Substitute ψ_n from eq^n (5)

$$\int_0^a B^2 \sin^2\left(\frac{n\pi}{a}\right)x . dx = 1.$$

But W.K.T $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$B^2\left[\frac{1}{2}\int_0^a dx - \frac{1}{2}\int_0^a \cos\left(\frac{2n\pi}{a}\right).x.dx\right] = 1.$$

$$\frac{B^2}{2}\left[x - \frac{a}{2n\pi}\sin\left(\frac{2n\pi x}{a}\right)\right]_0^a = 1.$$

$$\frac{B^2}{2}\left[a - \frac{a}{2n\pi}\sin\left(\frac{2n\pi \cancel{a}}{\cancel{a}}\right)\right] = 1 \quad (\because \sin(2\pi n) = 0)$$

$$\frac{B^2}{2}\left[a - \underbrace{a/2n\pi\ (0)}_{0}\right] = 1$$

$$Ba^2/2 = 1 \qquad \therefore B^2 = 2/a \qquad \therefore \boxed{B = \sqrt{\frac{2}{a}}}$$

Hence the normalised wave function of a particle in a one dimensional infinite potential well is given by

$$\boxed{\psi n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\right)x} \rightarrow 6$$

$$\therefore \boxed{E_n = \frac{n^2h^2}{8ma^2}}$$

This gives the energy eigen values of the particle in an infinite potential well.

The lowest acceptable value for 'n' is 1. consequently lowest energy corresponds to n=1. which is called "zero point Energy"

The zero point energy of an particle in an infinite potential well is given by

$$\boxed{E_{zero point} = \frac{h^2}{8ma^2}} \quad \text{where } n=1.$$

Note:

*. The lowest permitted state of energy is called "ground state energy".

*. The zero point energy is taken as the ground state energy.

*. The states of energy corresponding to n>1 are called "Excited states"

Calculate the wave functions (or) eigen function ($\psi_1 \cdot \psi_2 \cdot \psi_3 \cdots$) probability densities ($|\psi_1|^2, |\psi_2|^2, |\psi_3|^2 \cdots$) & energy levels (Eigen values) for a particle in an infinite potential well.

Let us consider the first three cases of eigen functions, probability densities & eigen energy levels (values) for the particle in the well by putting $n = 1, 2, 3 \cdots$

Case ①: when $n = 1$

This is the ground state & the particle is normally found in the ground state.

for $n=1$, The eigen function is

$$\psi_1 = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a}\right) x. \qquad \left(\because \psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}\right) x\right)$$

In the above eq^n, $\psi_1 = 0$ for both $x = 0$ & $x = a$.

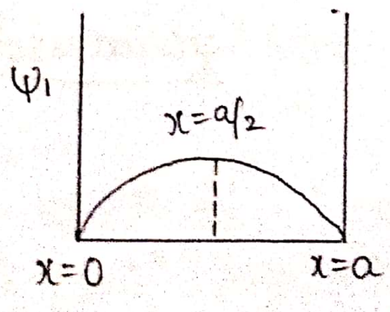
But ψ_1 is maximum value for $x = a/2$

$$\therefore \psi_1 = \sqrt{2/a} \sin(\pi/a)(a/2) \qquad (\because x = a/2)$$

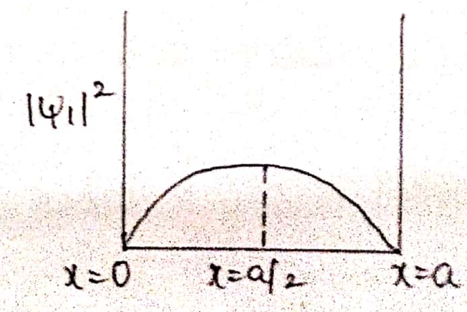
$$\boxed{\psi_1 = \sqrt{2/a} \sin(\pi/2)}$$

Hence plot the graph of ψ_1 (V/s) x & $|\psi_1|^2$ (V/s) x.

ψ_1 (V/s) x



$|\psi_1|^2$ (V/s) x



From the fig. $|\psi_1|^2 = 0$ at $x=0$ & $x=a$.

$|\psi_1|^2$ is maximum at $x = a/2$.

This means that particle cannot be found at the walls & the probability of finding the particle is maximum at $x = a/2$ (central region).

The energy of the particle is given by,

$$\boxed{E = \frac{n^2 h^2}{8ma^2}}$$

when $n = 1$, $\boxed{E_1 = \frac{h^2}{8ma^2}} = E_0$ or $\boxed{E_1 = E_0}$

This energy is known as Zero point energy

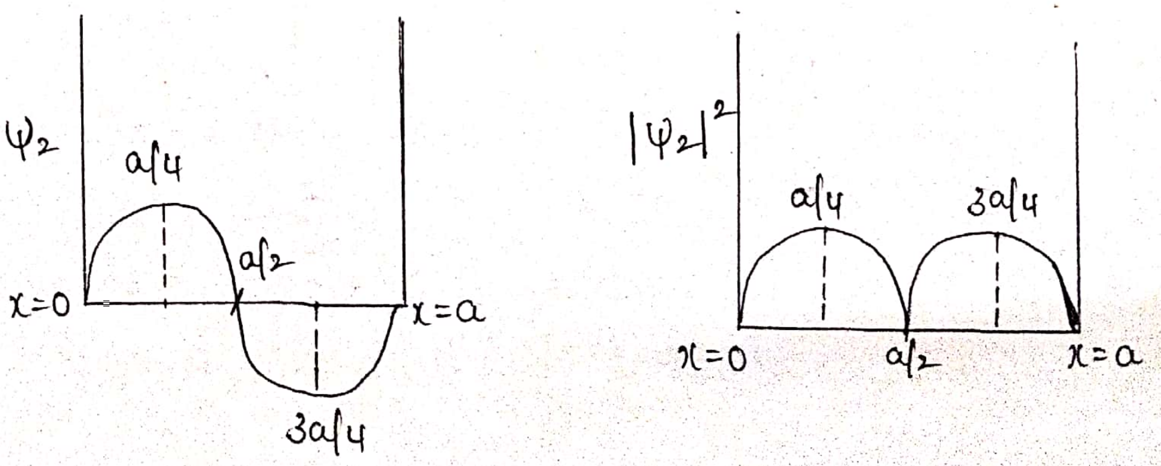
Case 2 : when $n=2$:

This is the first excited state. The eigen function for this state is $\boxed{\psi_2 = B \sin\left(\frac{2\pi}{a}\right) x}$

Now, $\psi_2 = 0$ for $x = 0$, $x = a/2$ & $x = a$.

ψ_2 is maximum for $x = a/4$ & $x = 3a/4$.

These facts are seen in the plot of ψ_2 (v/s) x & $|\psi_2|^2$ (v/s) x.



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It is seen that $|\psi_2|^2 = 0$ at $x=0$ & $a/2$ & a. It means that in the first excited state the particle cannot be observed either at the walls (or) at the center.

for $n=2$, $E_2 = \frac{4h^2}{8ma^2}$ $\left(\because E_1 = \frac{h^2}{8ma^2}\right)$

$$\boxed{E_2 = 4E_1}$$

Hence the energy in the first excited state is 4 times the zero point energy.

Case (3) : when $n=3$:

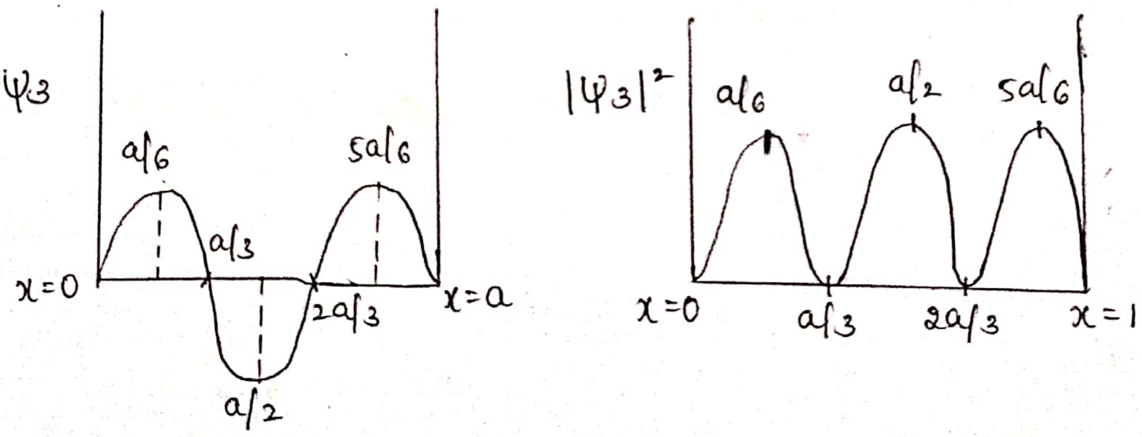
Eigen function for the second excited state is given by

$$\boxed{\psi_3 = B\sin\left(\frac{3\pi}{a}\right)x}$$

Now $\psi_3 = 0$ for $x=0$, $x=a/3$, $x=2a/3$ & $x=a$.

ψ_3 is maximum for $x=a/6$, $a/2$ & $5a/6$.

The plot of ψ_3 (v/s) x. & $|\psi_3|^2$ (v/s) x are in the fig.



The probability densities has maximum at $x = a/6$, $a/2$, & $5a/6$. which also implies the locations at which the particle is most likely to be found.

for $n=3$, $E_3 = \frac{(3)^2 h^2}{8ma^2}$

$$\Rightarrow \boxed{E_3 = \frac{9h^2}{8ma^2} = 9E_1}$$

The energy levels are discrete but not continous.

Eigen values for a free particle:

A free particle is one which is not under the influence of any kind of field (or) force & there is no boundary conditions. Hence $V=0$ holds good everywhere.

We can extend the case for particle in one dimensional infinite potential well to a free particle by taking the width of the well to ∞ $(a \to \infty)$

Equation for energy eigen values for a particle in an infinite well as;

$$E = \frac{n^2h^2}{8ma^2} \quad \text{where } n = 1,2,3 \ldots$$

$$n = \frac{2a}{h}\sqrt{2mE}$$

Now as $a \to \infty$, we have $n \to \infty$, This shows that a free particle can have any energy value & energy is continous. There is no quantization of energy & free particle becomes a "classical entity".